



DEVELOPING NUMBER SENSE IN IRISH PRIMARY SCHOOL CLASSROOMS

A Resource Booklet for Teachers

This PDST booklet is intended to support Irish primary school teachers in their development of number sense with pupils, particularly pupils in junior classes. It has been designed to accompany the PDST Number Sense mathematics workshops.

This resource has been designed by members of the Professional Development Service for Teachers. Its sole purpose is to enhance teaching and learning in Irish primary schools and will be mediated to practising teachers in the professional development setting. Thereafter it will be available as a free downloadable resource on www.pdst.ie for use in the classroom. This resource is strictly the intellectual property of PDST and it is not intended that it be made commercially available through publishers. All ideas, suggestions and activities remain the intellectual property of the authors (all ideas and activities that were sourced elsewhere and are not those of the authors are acknowledged throughout the resource). It is not permitted to use this manual for any purpose other than as a resource to enhance teaching and learning. Any queries related to its usage should be sent in writing to:

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What is number sense?

'Number sense' is sometimes referred to as having a 'feel for numbers' and refers to a pupil's facility with numbers that is underpinned by an awareness of the relationships between numbers, which can then support pupils in their interpretation of new number problems.

Number sense is about having a competency with numbers that is based on understanding rather than being reliant on memorisation, recall, or the use of algorithms. Pupils with number sense possess an ability to make generalisations about the patterns and processes they have met and then can link new information to their existing knowledge.¹

Number sense develops over time resulting from exploring numbers, visualising them in a variety of contexts, creating an awareness of number relationships, and solving mathematical problems that are not bound or limited to traditional algorithms.² Put simply, number sense is a good intuition about numbers, their relationships, and their magnitude.³

The components of number sense have been defined as⁴:

composing and decomposing numbers

recognising the relative magnitude of numbers

using benchmarks

linking representations

understanding the effects of operations

inventing strategies

estimating

possessing a disposition toward making sense of numbers

¹ Anghileri, 2000

² Sood & Jitendra, 2007; Bobis, 1996

³ Howden, 1989; Fennell & Landis, 1994

⁴ Sarama & Clements, 2009

Why is developing number sense important?

Number sense is critically important to pupils' mathematical development including pupil achievement in mathematics. For example, an important difference in relation to number sense has been found between low-achieving and high-achieving 7 to 13 year old pupils in mathematics. The research suggests that high achieving pupils use number sense and low achieving pupils do not. For example, in approaching problems such as $19 + 7$, the high achievers changed the problem into, for example, $20 + 6$ whilst no low achieving pupils used number sense. Similarly, in approaching subtraction problems such as $21 - 16$ the high achieving pupils used strategies such as changing the numbers into $20 - 15$, whilst the low achieving pupils counted backwards starting at 21 and counting back 15—a very difficult task. The researchers concluded that low achievers are often low achievers not because they know less but because they don't use numbers flexibly. Instead, they have been set on the wrong path, often from an early age, of trying to memorise methods instead of interacting with numbers flexibly.⁵

This emphasis on memorisation at the expense of an understanding of, and flexibility with numbers, can inhibit pupils' mathematical development. So, "the more we emphasize memorization to students the less willing they become to think about numbers and their relations and to use and develop number sense".⁶ Another study supports this finding—a study involving 13 million students worldwide found that students who take a memorisation approach in mathematics are the lowest achieving students in the world.⁷ Therefore, when pupils develop number sense and number flexibility, they build the most important foundation for all higher levels of mathematics.

Although mathematics facts have a role to play in mathematical development, they should be learned in a way that promotes understanding and flexibility, thus ensuring a more efficient way to 'remember and recall' than merely learning by rote. A memorised fact or procedure demands no higher order skills and often results in pupils who may be able to apply a memorised procedure but cannot justify why their solution is reasonable.⁸ This is at odds with the The Irish Primary School Mathematics Curriculum where Junior and Senior Infant pupils are expected to "justify the process or results of activities" (p.18) and 1st and 2nd

⁵ Gray & Tall, 1994

⁶ Boaler—accessed from <https://www.youcubed.org/fluency-without-fear/>

⁷ Boaler & Zoido, 2016

⁸ Parish, 2010

class pupils are expected to “justify the processes and results of mathematical activities” (p.28).

The need for pupils to understand, not just recall, number facts—a key tenet of number sense—is reinforced in the Irish Primary School Mathematics Curriculum (1999, p.39) in which the understanding of number facts is emphasised, alongside that of recall, for 1st and 2nd classes, as is the need for pupils to “recognise and create mathematical patterns and relationships” (p.38). Similarly, instead of merely recalling number facts, pupils in Infant classes and 1st and 2nd classes are expected to “devise and use mental strategies and procedures for carrying out mathematical tasks” (p.18/39).

How can number sense be supported and enhanced?

Number sense can and should be taught and so number sense can be developed with all pupils in the class. There are many ways in which number sense can be supported and developed in classrooms including:

- counting
- learning experiences aligned with those in the Ready, Set, Go Maths programme for infant classes;
- PDST support material for developing a facility with, and understanding of, number;
- using models and tools in representing numbers; and
- using ‘number talks’ as a classroom strategy.

Counting

A facility with counting provides a strong foundation for the development of number sense. Counting allows pupils to count on and count back, which represents an important first step in mental calculation. At a more sophisticated level, skip-counting can be used to further develop number sense as can bridging through ten⁹ with larger numbers, for example, through 100, through 1000, through 10,000, etc. The importance of flexibility in pupil’s counting cannot be over-emphasised. There are three basic approaches that help pupils develop this flexibility:

- **Different starting points:**
Young pupils can typically recite the names of numbers in sequence as if reciting a rhyme, however, starting from a different number (e.g. not starting from 0 or 1) requires an understanding of number relationships, not just recitation, and so can contribute to developing flexibility in counting.
- **Counting backwards:**
While the forward counting sequence is typically familiar to most pupils, counting backwards can be more challenging and requires a more robust understanding of number relationships, thus contributing to the development of flexibility in counting.

⁹ Bridging through ten is sometimes also referred to as Crossing the ten.

- **Crossing the 10 (or the 5):**

Once pupils can securely count forwards to 5, then 10, it is important to count beyond the 5/10 in order to reinforce the idea that counting is continuous, does not stop at 10, and can be start and stop at various numbers. Furthermore, crossing the 10 can be challenging for pupils because what comes after 10 can be different to the word pattern that precedes it (e.g. counting on from 17 to 23 requires pupils to change from counting using a teen pattern—seventeen, eighteen, nineteen—to counting using a different format—twenty, twenty-one, twenty-two, twenty-three, etc.). This challenge is particularly acute when crossing multiples of 10. Regular practice crossing the 10 from various starting points—both counting forwards and backwards—can also contribute to flexibility in counting.

Ideas for developing counting in classrooms are outlined on the PDST website¹⁰ and can be accessed [here](#).

Ready Set Go Maths

Learning opportunities outlined in Ready Set Go Maths¹¹ (Eunice Pitt)—for example, sorting, combining and partitioning sets, counting and recognition, understanding number, developing number relationships and operations—contribute to the early development of number sense. Supporting materials developed by PDST for using Ready, Set, Go Maths can be downloaded for free from <http://www.pdst.ie/earlymaths>.



¹⁰ http://www.pdst.ie/sites/default/files/Counting_activities.pdf.

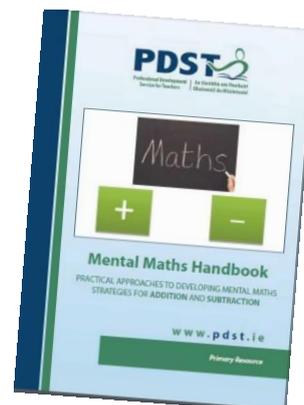
¹¹ Ready, Set, Go Maths manuals can be purchased from Northern Ireland at https://ccea.org.uk/curriculum/foundation_stage/areas_learning/mathematics_and_numeracy/ready_set_go

PDST Support Material

PDST have developed support material that can also be used to contribute to the development of number sense. The support material most relevant for developing number sense are the Mental Maths Handbook and the Place Value, Decimals, and Percentages Teacher's Handbook

Mental Maths Handbook

The Mental Maths Handbook is intended to support teachers in developing addition and subtraction mental maths strategies in their classrooms. The material explores the key properties of number and number relationships relating to addition and subtraction. It also explores background knowledge for teachers and fundamental facts in relation to mental maths. A range of concrete, practical activities that will support pupils in their development of addition and subtraction mental maths strategies is also outlined. Finally, a selection of engaging and enjoyable activities to consolidate learning and provide opportunities for pupils to master addition and subtraction facts is included. The strategies outlined in the support material include:



Counting forwards & backwards, counting on

Making 5/10

Doubles/ Near doubles

Bridging through tens

Reordering

Partitioning by place value

Compensating

Subtraction: as think addition (shopkeepers method) and keeping constant difference

In addition to developing number sense, mental computation is a key component of number talks because it encourages pupils to build on number relationships and to solve problems instead of relying on memorised procedures. One of the purposes of number talks is for pupils to focus on number relationships and to use these relationships to develop efficient,

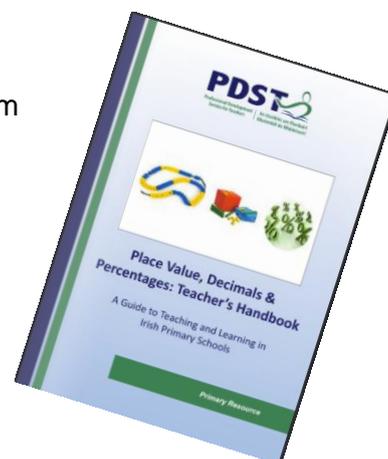
flexible strategies with accuracy. When pupils are encouraged to approach problems without paper and pencil they must rely on what they know and understand about the numbers and how they are interrelated. This Mental Maths Handbook can be downloaded for free from the PDST website.¹² Click [here](#) to access this support material.

Place Value, Decimals & Percentages: Teacher's Handbook

The aim of this support material is to assist teachers in teaching the strand units of Place Value (1st to 6th class), Decimals (3rd to 6th class) and Percentages (5th and 6th class). The learning experiences outlined for Place Value are particularly relevant for developing number sense, for example, the hundred chart/square helps students count, model addition and subtraction strategies, and notice important ideas and patterns about the base ten system.

The approach outlined in this support material advocates moving from the concrete to the pictorial to the abstract, based on the needs of individual pupils. Representing numbers in a variety of ways has been shown to increase flexibility with, and understanding of, numbers thus contributing to the development of number sense.

In addition to exploring suitable learning experiences for pupils, the handbook outlines Background Knowledge for Teachers including fundamental facts and possible misconceptions and challenges that pupils may face when developing place value. For example, one of the challenges in developing place value with pupils mirrored as a challenge in developing number sense:



Some pupils have difficulty in understanding the different conventions which are necessary to say/read the numbers between 11 and 19. Much discussion will be necessary for these pupils in relation to the names of numbers.

The Place Value, Decimals, and Percentages Teacher's Handbook can be downloaded for free from the PSDT website.¹³ Click [here](#) to access this support material.

¹² <http://www.pdst.ie/sites/default/files/Mental%20Maths%20Workshop%201%20Handbook.pdf>

¹³ [http://cmsnew.pdst.ie/sites/default/files/A%20Guide%20to%20Teaching%20Place%20Value%20Per centages%20and%20Decimals_printing%20version_1.pdf](http://cmsnew.pdst.ie/sites/default/files/A%20Guide%20to%20Teaching%20Place%20Value%20Percentages%20and%20Decimals_printing%20version_1.pdf)

Multiple Representations of Number

A variety of models to represent number can be useful in developing number sense with pupils. Much research attests that ‘multiple representations’ in maths are important as they serve to deepen mathematical understanding of concepts.¹⁴ It is important to provide opportunities for using a varied range of models to represent number, and number relationships because:

- different models may resonate with different pupils allowing some to ‘get it’ with one model and not another
- the use of many models will deepen the conceptual understanding of learners
- different models may appeal to different learning styles and so suit some pupils more than others



It is important to provide pupils with regular opportunities for representing number, and number relationships, in a variety of ways using a variety of models. These should include verbal, concrete, pictorial and symbolic models of representing number.

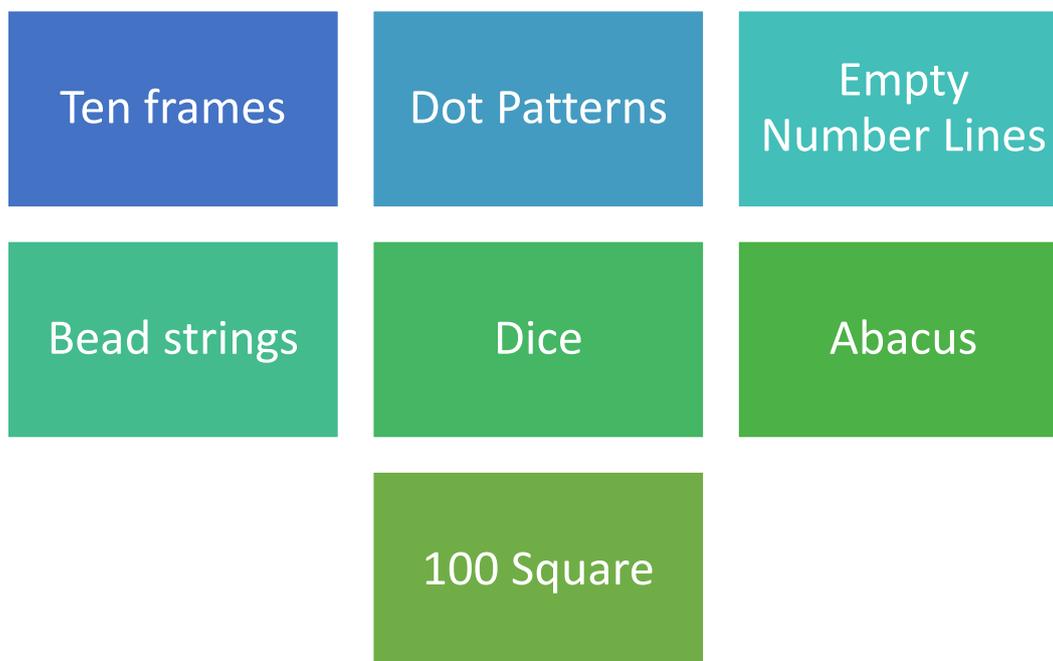
Furthermore, the Irish Primary School Mathematics Curriculum emphasises the importance of using multiple ways to represent number. For example, Junior and Senior Infant classes as well as 1st and 2nd classes are expected to develop the skill of recognising “...the relationship between verbal, concrete, pictorial and symbolic modes of representing numbers” (p.18/38).

Many models are based upon recognising pattern and subitising. Subitising is a fundamental skill in the development of pupils’ understanding of number¹⁵. It refers to the ability to be able to ‘see’ a number, without actually counting each individual dot, for example, when a pattern is flashed up for a few seconds you can determine the number by associating it with number patterns that you already know. Subitising is a complex skill that needs to be developed and practised through experiences with patterned sets.

¹⁴ Duncan, 2010; Kaput et al, 2002; Roschelle et al, 2000; Hegedus & Kaput, 2007

¹⁵ Baroody, 1987

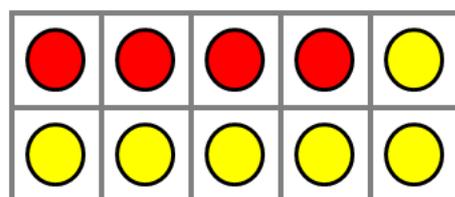
Concrete and pictorial models that may be suitable for representing number, and number relationships, from Junior Infant to Second classes might include (but are not restricted to):



The first three of these (ten frames, dot patterns, and empty number lines) are detailed below. A printable version of a 100 Square is provided in Appendix A.

Ten Frames

Ten frames can be used concretely—using counters or cubes on a ten frame—and pictorially—colouring in a ten frame template. Five and ten frames can be used as a single row of five (five frame) or two rows of five (ten frame) or by putting two ten frames side-by-side so that pupils can work with numbers up to 20. Using two ten frames in this way is a useful strategy for pupils to explore bridging (or crossing) the ten (using different colours can make this more explicit for pupils).



Ten frames can be used to foster fluency, practice subitising, explore place value, and compute using addition and subtraction. Using an empty frame to represent zero is important. Frames are also arranged to capitalise on subitising to five as half of ten. Children should first build fluency 1-6 and then 7-10.

Possible questions when using ten frames might include:

- How many did you see?
- How do you know?
- How did you see seven?
- How many 'blanks' were there when you saw seven?
- What does it tell us about seven?
- What does this tell us about ten?
- How many more do we need to make ten?
- How many are left after removing three?

It is important to focus on open rather than closed questions. Five and ten frames can also be used to show the commutative property by turning the frame upside down, for example, '3 and 2 equals 5 and if I turn the frame upside down, then 2 and 3 equals 5 also.'

As well as contributing to the development of number sense, ten frames can be used to:

examine the underlying properties of odd and even numbers

explore the commutative property

explore the associative property

develop and consolidate addition (including bridging through 10)

develop and consolidate subtraction (including subtraction as deducting, complementing, difference)

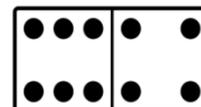
introduce and reinforce the inverse relationship between addition and subtraction

explore early multiplication and division.

Templates for ten frames can be found in Appendix B.

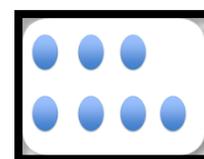
Dot Patterns (including dominoes, dot card strings, and Hungarian number pictures)

Just like with a standard dice, instant recognition can be developed for other patterns. Providing opportunities to build dot pattern recognition with pupils, is working *with* the brain because our brains seek out pattern and try to make sense of it—dot patterns utilise this facility. Using two colours supports early learners to recognise that some patterns are made up of two smaller patterns or one or two additional dots.

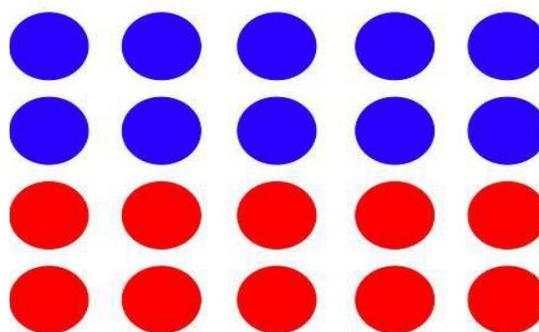
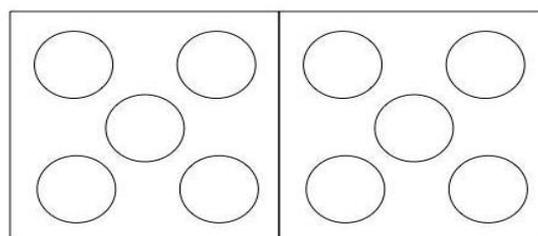


Some key teacher questions include:

- How many dots do you see?
- How did you see them?
- Did anyone see them a different way?
- What does this tell us about the number X?
- What if I turn the card/dominoe this way? Now what do we know about the number?
 (Rotate the card 180 degrees to demonstrate the commutative property.)



Hungarian Number Pictures are used by teachers in Hungary as a way of supporting pupil's understanding of numbers and their learning of number bonds to twenty. They are based on the notion that we can instantly recognise numbers to five without counting (a skill called subitising) and so arranging the number of dots in the holes on the template help.



Different numbers can be shown in different colour dots for addition. In Hungary, pupils have counters that are blue on one side and red on the other. Two sets will enable you to work with the numbers to twenty. The pictorial representation helps pupils to remember and visualise the number bonds.¹⁶ A template for Hungarian Number Pictures can be found in Appendix C.

¹⁶ Adapted from www.nrich.maths.org/8119

Empty Number Lines

An empty number line is a powerful tool which can be used in the development of number sense. It is particularly easy to use because all that is required is something to write on, for example, a pencil and piece of paper, a blackboard and chalk, a whiteboard and marker, etc. The pupils draw a straight line—the empty number line. The pupil can then use this line to ‘record’ his/her thoughts or steps in a calculation. In the image below, the calculation $8 + 9$ is represented on an empty number line. In this instance, a number of steps are taken:

- What goes with 8 to make 10?
- Hence, 9 is broken down into 2 and 7
- 8 is written as the starting point
- 2 is added to 8 to make 10
- 7 is added to 10 to make 17



The empty number line is a useful way of jotting down or informally recording these steps. In fact, this calculation could be solved in a slightly simpler way by starting with the bigger number, for example:

- What goes with 9 to make 10?
- Hence, 8 is broken down into 1 and 7
- 9 is written as the starting point
- 1 is added to 9 to make 10
- 7 is added to 10 to make 17

Starting with the bigger number becomes even more efficient when two numbers vary greatly in size, for example, $18 + 7$.

Number Talks

Number talks is a teaching strategy that comprises classroom conversations around purposefully crafted computation problems that are solved mentally. Carefully crafted number talks are a powerful teaching and learning strategy in developing number sense. However, it is important to note that using number talks effectively does not just contribute to developing number sense. The following have been proposed as goals for number talks:¹⁷

1. Develop number sense
2. Develop fluency with small numbers
3. Subitising
4. Making tens

Number talks was developed by Ruth Parker and Kathy Richardson and is a useful short teaching activity (typically 5 to 15 minutes) with which teachers can begin lessons or parents can do at home. Number talks work well as a whole class activity and also works well in groups. It involves:

the teacher posing an abstract purposeful maths problem such as 18×5 and asking pupils to solve the problem mentally

pupils exercising numerical reasoning by finding a solution mentally

the teacher collecting the different methods and exploring with pupils why different methods work

pupils sharing and defending solutions and strategies

pupils collectively reasoning about numbers

pupils discussing which method might be the most efficient

¹⁷ Parrish, 2010

The problems chosen for a number talk are designed to elicit specific strategies that focus on number relationships and number theory. The teacher's goals and purposes for the number talks determine the numbers and operations that are chosen. A mixture of random problems do not lend themselves to a common strategy. While they might be used as practice for mental computation they do not initiate a common focus for a number talk discussion. Although crafting problems with a specific strategy in mind, it is important to bear in mind that pupils may, and can, use other strategies. Numerous strategies exist for any one problem; however, specific types of problems typically elicit certain strategies, for example, 19×4 is crafted to target pupil's thinking using tens as friendly or landmark numbers. So, if trying to elicit the strategy 'think addition' (adding up/ shop keepers method), then the following types of problems might be used: $23 - 19$; $23 - 16$; $23 - 14$; $23 - 9$, etc.



Eight different strategies for developing mental addition and subtraction are outlined in the PDST support material Mental Maths Handbook (referenced on page 7).

Multiple Solutions

In number talks, pupils are expected to solve problems accurately, efficiently and flexibly. Efficiency refers to the ability to choose an appropriate expedient strategy for a specific computational problem. During a number talk, the teacher writes a problem on the board and gives pupils time to solve it mentally. Pupils start with their fists to their chests and indicate when they are ready with a solution by raising a thumb. Once they have found an answer they are encouraged to continue finding efficient strategies while others are still thinking. Pupils indicate that they have other strategies by raising another finger for each new strategy. This allows other pupils time to think while challenging those who already have an answer. When most pupils have a solution answer, the teacher invites solutions and records these, whether correct or incorrect, on the board for all to consider. Pupils then share their strategies and justifications.¹⁸ Pupils can then be encouraged to vote for the strategy which they consider to be most efficient, giving reasons for their choice.

¹⁸ Parrish, 2014

By sharing and defending their solutions and strategies, pupils have the opportunity to collectively reason about numbers while building connections to key conceptual ideas in mathematics. In sharing and discussing computation strategies pupils can:

- clarify their own thinking
- consider and test each other's strategies to investigate whether they are mathematically logical
- investigate and apply mathematical relationships
- build a repertoire of efficient strategies
- make decisions about choosing efficient strategies for specific problems.¹⁹

¹⁹ Parish, 2010

Further Reading/Viewing

Number Sense

Video

What is number sense? (Jo Boaler) at <https://www.youcubed.org/what-is-number-sense/>

Website

Number Sense Series: Developing Early Number Sense at <https://nrich.maths.org/2477>

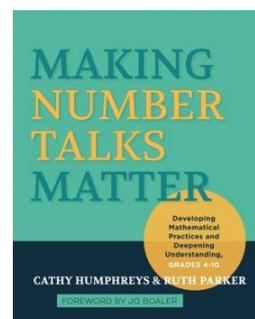
Number Talks

Book

Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding (Cathy Humphreys and Ruth Parker)

Video

A video clip of Cathy Humphreys and Ruth Parker discussing number talks at <https://www.youtube.com/watch?v=Hmh8pwL1sLg>

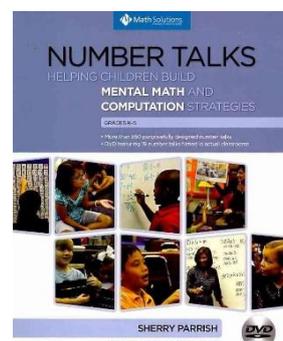


Book

Number Talks: Helping Children Build Mental Math and Computation Strategies (Sherry Parrish)

Video

A video clip of Sherry Parrish discussing Number Talks: Building Numerical Reasoning at <https://www.youtube.com/watch?v=twGipANclgq>

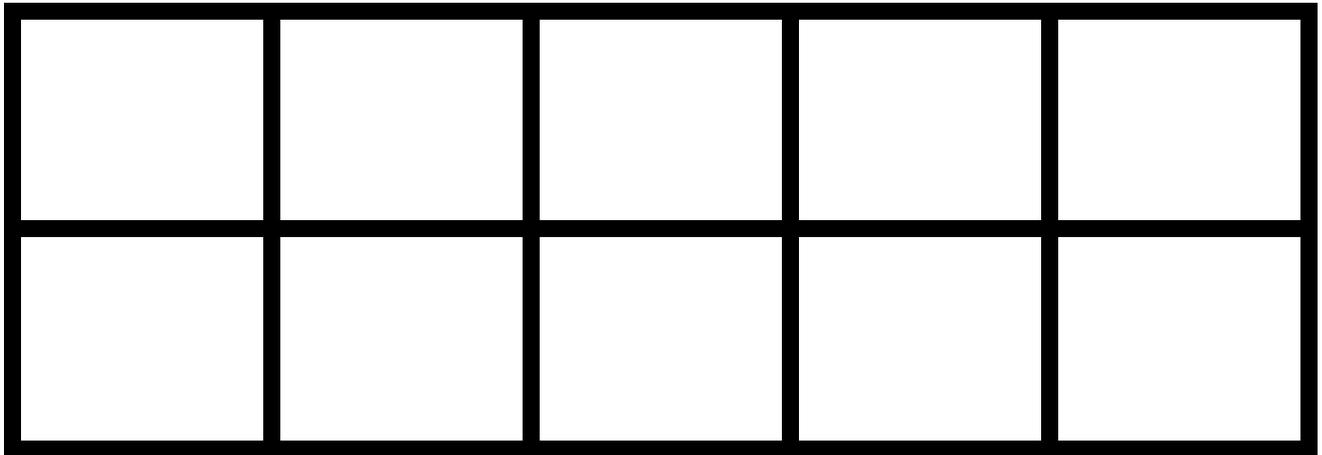
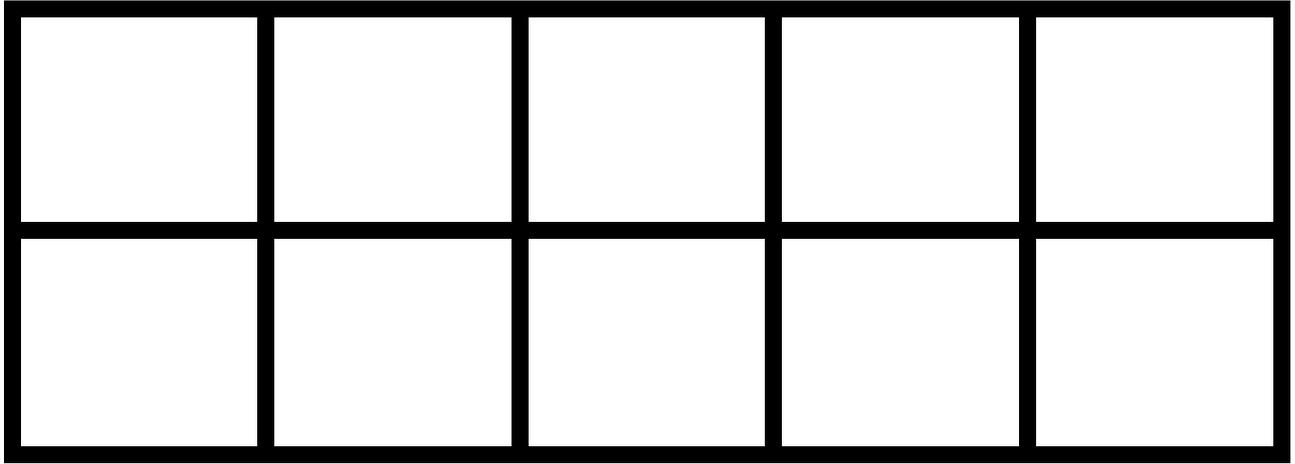


Appendices

Appendix A: 100 Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Appendix B: Templates for Ten Frames



Appendix C: Templates for Hungarian Number Pictures

