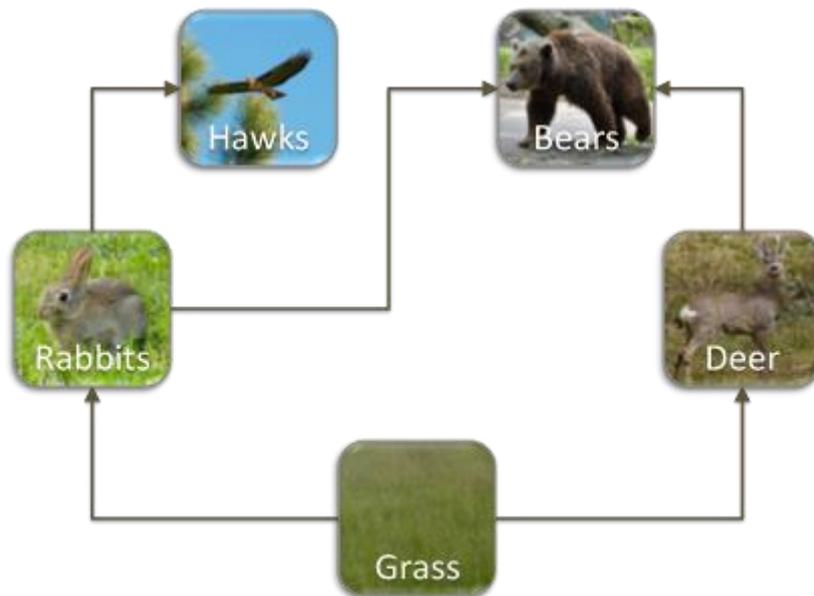


Q1. Food Web



The image above shows a simple food web for this eco-system. Rabbits and deer eat grass, so there is a transfer of energy from the grass to them, indicated by the arrows. Hawks eat rabbits, and bears eat both rabbits and deer, similarly indicated by arrows.

If the deer started to reproduce rapidly, there is probably no danger of there being insufficient grass, since there is ample grass in the ecosystem. However, more deer would mean there was a plentiful supply of food for the bears, so they would also increase in number.

If enough rabbits migrate, there would be insufficient food for the hawk population, leading to a decrease in hawk numbers. If there was a simultaneous decrease in the deer population, then the bear population would be seriously threatened.

If the bear were hunted by humans, they would be in severe danger of extinction in that area, given that there are only 5 of them. This would be likely to result in an increase in both the deer and rabbit populations, and hence an increase in the hawk population.

In the autumn, the 5 bears would need 200kg of food per day. This is equivalent to 200 rabbits a day, or 4 deer every 3 days, or a combination of both.

The 10 hawks would need 10 kg of food several times a week, say around

40kg per week, or around 6kg per day. This is equivalent to 6 rabbits a day.

If we ignore the deer, then 206 rabbits a day are eaten by the bears and the hawks. At this rate, the 2000 rabbits will be gone in less than 10 days, and the bears and hawks will then also die.

50 deer (without any rabbits) would provide food for the bears for about 37 or 38 days.

However, the rabbits will reproduce quickly. If the 2000 rabbits are half male and half female, then 1000 females could produce up to 800,000 rabbits in a season, if there were no predators. This is certainly enough food for the hawks, and would also provide more than enough food for the bears, provided they did not eat too many rabbits too soon. The addition of some deer would probably ensure that this did not happen.

Source(<http://nrich.maths.org/7592/solution>)

1. Yes - As $1\text{m}=100\text{cm}$ a crate with dimensions 100cm by 50cm by 50cm will have volume $V=250,000\text{cm}^3$. Hence, the number of water bottles required to fill this up is

$$N=250,000/500=500$$

2. Yes - If the diameter of the tree is almost half a meter, its radius is almost 25 cm . Now, if the tree's radius grows by 0.6 cm in each year, in 42 years the radius should be about $42 \cdot 0.6 = 25.2\text{cm}$, and this is about right. However, to make a safer estimate (as it is unlikely that the tree will be growing by exactly 0.6 cm each year) a range of the form $38 - 45$ years would be better.

3. No - While $365 \cdot 0.3 = 10.95\text{kg}$, which is close to the estimate, perhaps it would be sensible to consider that the scientist only has crisps on the *working days* of the year (since she is having a pack in her morning break). Now, a person is on average working 44 weeks each year, so they have $44 \cdot 5 = 220$ working days (and perhaps slightly less, if we take into account bank holidays).

Therefore, the scientist is more likely to be consuming $220 \cdot 0.3 = 6.6\text{kg}$ of crisps each year.

4. No - A teaspoon of sugar weighs about 5 grams. So, the scientist and her friend consume together 8 cups of coffee each day, with 16 teaspoons of sugar. So, each day they consume $16 \cdot 5 = 80\text{g}$ of sugar. Thus, in a two - month period, they would need $60 \cdot 80 = 4800\text{g}$ of sugar, which is much more than a packet!

5. No - Once again, the calculation $365 \cdot 22 = 8030$ is correct, but 365 is not the right number to use, as she is not working every day of the year. Using our previous estimate for the number of working days in a year, we see that the scientist is actually going to claim around $220 \cdot 22 = 4840$ miles from work.

6. Yes - In this case, it is highly likely that the lab will be working all year round, so the estimate $560 \cdot 12 = 6720$ is about right.

7. Yes - We can model the rubble by a large cone, whose height is 2m and base radius is 1.5m. The volume of such a cone is

$$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h = \frac{1}{3} \cdot \pi \cdot 1.5^2 \cdot 2 \approx 4.7 m^3$$

So the estimation is correct, and a 5 cubic meter skip should be large enough.

8. No - $124 \cdot 170 = 21080 m^3$ mean that each minute there are 21080 cubic meters of water flowing. Hence, over the course of an hour, there will be $21080 \cdot 60 = 1,264,800 m^3$ flowing, which is equivalent to 1,264.8 metric tonnes.

9. Yes - The area of the vegetable plot is $A = 9.5 \cdot 11 = 104.5 m^2$. Now, with the test data as described, we expect that *on average* there are $42 + 532 = 47.5$ worms per square meter.

Hence, a good estimate for the number of worms in the whole plot is $N = 47.5 \cdot 104.5 = 4963.8$, so the scientist is correct in this case.

<http://nrich.maths.org/7418/solution>

Time

That a pet rat lives **2-3 years**

For all of the red blood cells in your body to be replaced....10 times **3 years**

Between two football world cups **4 years**

For light to reach us from the nearest star other than the sun **4.2 years**

Distance

You can hop in 5 seconds **8 metres**

The distance from the penalty spot to the goal on a football field **11 metres**

From the top of a 6 storey building to the ground **18 metres**

A car can drive on 1cm^3 of fuel **20 metres**

Speed

Of the fastest sprinter in the world **10m/s**

Of a bus going past the school (30mph = **13.4m/s**)

Of a racehorse running at maximum speed **15m/s**

Of the fastest cyclist in an Olympic cycle sprint race **19m/s**

Mass

Of enough potatoes to make chips to feed everyone in the school for a week.
(**1000kg** for 1000 students)

Of a million cubic centimetres of water **1000kg**

Of a standard family car **1500kg**

Of a team of international male rugby players (15 players) **1500kg**

Volume of water

used in a 10 minute shower **90 litres**

used in 5 dishwasher cycles **100 litres**

in a half-filled bath **100 litres**

used to flush the toilet 20 times **120 litres** (modern toilet)

Number of

Steps when walking at a leisurely pace for 1 hour **8000**

Mars bars with the same total weight as your whole class **20000** (for a class of 30 students weighing about 40kg each)

Stitches in a handknitted jumper **75000**

People at a cup final in a large stadium **80000**

Area

Of the sheets of paper in a ream (500 sheets) of A4. **30m²**

Covered by 5 litres of paint **50m²**

Of the floor of the classroom **60m²**

Of fabric needed to make school uniforms for the whole class **will depend on the size of the class, the width of the fabric and the design of the uniform!**

<http://nrich.maths.org/7500/solution>

1. How many breaths will you take in a lifetime?

You might notice that you breathe once every 5 seconds. This means 12 breaths per minute, which is 720 breaths per hour, or 17280 breaths in a whole day.

Now, the average person lives 80 years, i.e. $80 \cdot 365 = 29200$ days (or 29220 days if you want to count the 20 leap years in between!)

So, the number of breaths you will **on average** take in your lifetime will be $29200 \cdot 17280 = 504576000!$

Of course, you need to take into account the fact that babies usually breathe faster, and also that during or after physical exercise you inhale and exhale a faster as well. This could certainly increase our estimation by a considerable amount.

2. How many people live within 1km of your school?

The area concerned is a circle centered at your school, with radius 1km - it is quite straightforward to see that this area is $\pi \text{km}^2 \approx 3.14 \text{km}^2$.

A simple assumption to make here is to consider the average population density of the UK (242 people per km^2) and multiply this by π . However, this is a relatively poor estimation. What you want to consider is the *urban* population density of the UK, which is obviously much higher (in fact 4348 people per km^2).

Then, an estimation of the number of people living within 1km of your school would be

$$3.14 \cdot 4348 = 13653$$

3. Volume of a cell.

If the cross-section of the cell is roughly circular, and we assume that the shape of the cell is spherical, then its radius is 3.5 microns - so its volume will be

$$V \approx \pi \cdot 3.5^3 \mu\text{m}^3 \approx 10.99 \mu\text{m}^3$$

4. How far does the Arctic Tern fly in its lifetime?

An arctic tern lives for about 30 years (being one of the longest living birds). Every year, it migrates from Arctica to Antartica in order to see two summers, and hence more daylight. The distance between Arctica and Antartica is 19000 km, so two such journeys a year mean that the Tern flies 38000 km per year, and hence 1,140,000 km in its entire life!

In fact, though, the Arctic Tern does not fly in straight lines, but rather along the coasts, so recent research shows it flies up to 70000km each year! See http://en.wikipedia.org/wiki/Arctic_Tern for more.

5. How much dog food is eaten in England each year?

First, we will calculate how many household dogs there are. It is estimated that one out of four households in the UK owns a pet (think about how many of your close friends own one in order to convince yourself). Also, given that there are approximately 17 milion families in England, we see that there are on average 4.25 milion domestic dogs.

Now, let's assume that an average dog needs a 500g can of food each day (of course there are several alternatives, such as dry food, and also it is important to remember that the amount of food greatly varies with the dog's age, type and weight).

Then, under that assumption, English dogs consume

$$4.25 \cdot 0.5 \cdot 365 = 775,625,000 \text{ kgs}$$

of dog food each year!

6. How long will it take for an amoeba to cover a petri dish?

A standard petri dish would have an area of about 100cm².

The amoeba, covers initially an area of 1mm², and it doubles every 24 minutes. This means that after k periods of 24 minutes, the area of the amoeba will be

$$A_k = 2^k \text{ mm}^2$$

Now, 100cm² = 10⁴mm².

We wish to calculate k for which $2^k > 10^4$. By considering various powers of 2, we see that $2^{13} = 8192$ and $2^{14} = 16384$. So, we require 14 periods of 24 minutes, or (less than) 5 hours and 36 minutes, for the amoeba to cover the entire petri dish.

7. How many cells do you have in your little finger?

We will assume that the human cells occupy an area of about $10^8 \mu\text{m}^2$. Now, your little finger should be about 5 cm long and 1 cm wide, so its area is 5 cm^2 . So, we divide the two to obtain that the number of cells in your little finger is about

$$N = \frac{5 \cdot 10^8 \mu\text{m}^2}{10 \mu\text{m}^2} = 5 \cdot 10^7$$

8. Estimate the weight of all your classmates' pets.

We expect that 1 classmate out of 4 owns a dog and 1 classmate out of 4 owns a cat. We also estimate that an average dog weighs 25 kg (although the range of possible dog weights is huge, so this estimation is not very well-founded) and that an average cat weighs 5 kg (the weights of cats are a lot more concentrated around this value).

Therefore, if your class has 20 pupils, the total weight of all your classmate's pets should be

$$W = 5 \cdot 5 + 5 \cdot 25 = 150 \text{ kg}$$

9. How much does your brain weigh?

In general, the brain of a newborn baby weighs about 300 grams (so is about 5% of the baby's entire weight). As the human is growing older, though, the brain's weight becomes a smaller percentage - in particular, an adult's brain weighs approximately 1,5 kgs, which is around 2% of the whole body's weight.

10. What will be the UK's population in 10 years' time?

The UK's population in 2001 was 58.8 million. The census of 2011, which was only recently carried out has not produced official results yet, but it is expected that the UK's population will appear to be around 62.4 million.

Now, if we are to assume that the population grows at a constant rate, we see that it should increase by 3.6 million every 10 years, so we would expect that in 2021 the UK's population should be approximately 66 million people.

However, over the course of 10 years, many factors which affect a country's demographics change, such as the economic situation in the UK and abroad, the likelihood of finding employment compared to other countries, environmental issues etc. So, the rate of change of the population is almost never the same over two decades. In fact, the tendency is that the rate of change increases, (this is what has been happening from 1981 onwards) and so a better estimate would be that the UK's population would significantly exceed 66 million by 2021.

(<http://nrich.maths.org/7275/solution>)