

Kruskal's Algorithm

1. To begin, select the edge of least weight.
2. Find the next edge of least weight. If it would form a cycle with the edges already selected, don't choose it. If not then add it to the MST.
3. If there is a choice of equal edges, it has no effect which you choose first.
4. Repeat step 2 until all vertices are connected.

Note: Kruskal's algorithm is a greedy algorithm which is where it builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.

The **purpose** of Kruskal's Algorithm is to find a subset of the edges that forms a tree and includes every vertex where the total weight of all of the edges is a minimum.

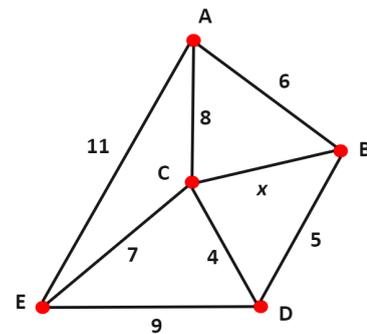
Kruskal's algorithm is most suitable for sparse graphs (low number of edges).

Additional Questions

Question 1

Seán's local football team is in the county final and his housing estate wants to show their support by lining the roads with bunting. They want each house to have bunting but they want to use the least amount of it.

- (a) The number on each edge below represents the distance, in metres, between each house. If $x = 4$, How much bunting do they need?
- (b) What could the variable, x , represent in a different scenario? What might affect it?

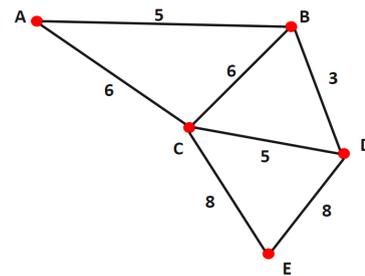


Question 2

In New York, there is a park with famous statues that has many visitors each day. Lighting is to be installed at 5 places in the park (as shown) with the places being connected either directly or indirectly by cabling following the given paths.

The values on each edge represent the distance (in m) between each light.

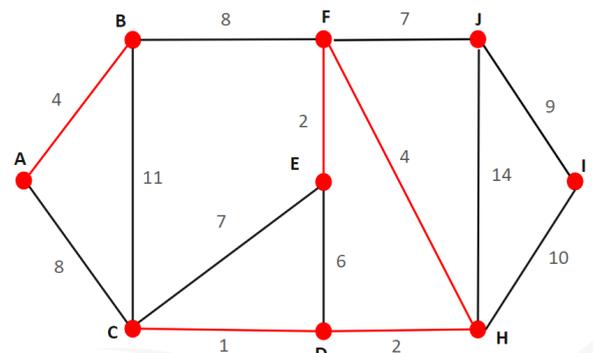
- (a) Calculate the shortest length of cabling required and show the minimum spanning tree (MST).
- (b) State two differences between Kruskal's algorithm and Prim's algorithm for finding a minimum spanning tree.



Question 3

For the network below, a student has commenced finding the minimum spanning tree (MST) using Kruskal's algorithm. The student's work is highlighted in red.

Is the student's work correct so far? If not, please correct it. Complete the MST to determine the weight of the tree.

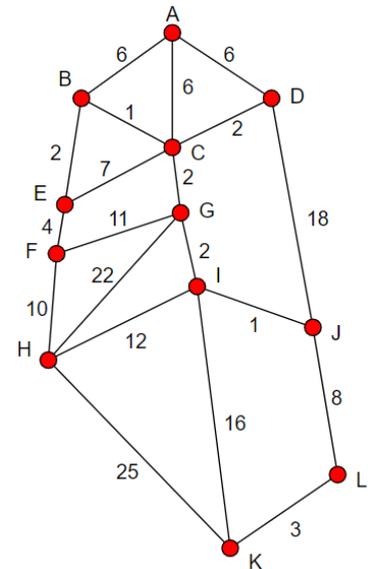


Question 4

- (a) When would you choose to use Kruskal's algorithm over Prim's algorithm?
- (b) Draw a network in which:
 - (i) The three shortest edges form part of the minimum spanning tree.
 - (ii) Not all of the three shortest edges form part of the minimum spanning tree.

Question 5

- (a) What is meant by the term 'greedy algorithm'?
- (b) A new broadband company wishes to lay new fibre optic cables in County Cork so that every house has access to broadband. It costs €350 per km to lay the cable. Calculate the total cost of the project. Each weight on the diagram represents distance in km.



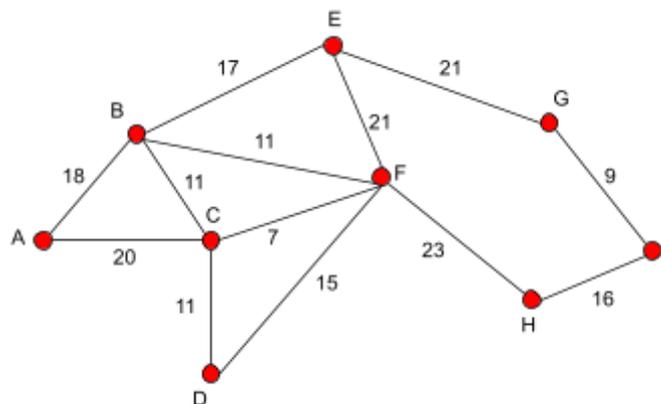
Question 6

Your local area is having a new broadband network installed. The initial phase is to first connect the post office, a shop, a sports club, primary school and secondary school.

- (a) Using Google Maps (or another mapping tool), create a network with the locations identified by nodes and the roads and distances represented by edges and weights.
- (b) Using Kruskal's algorithm, identify the length of cabling required and show the minimum spanning tree.

Question 7

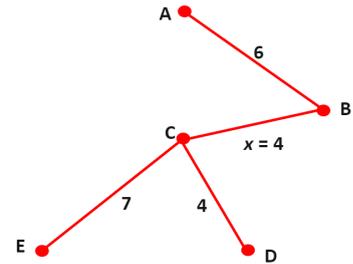
- (a) Does Kruskal's algorithm begin by selecting an edge or node?
- (b) The diagram below represents a network of paths in a park. The number on each edge represents the length of the path in metres. Using Kruskal's algorithm, find a minimum spanning tree for the network in the diagram and state its total length.



Solutions:

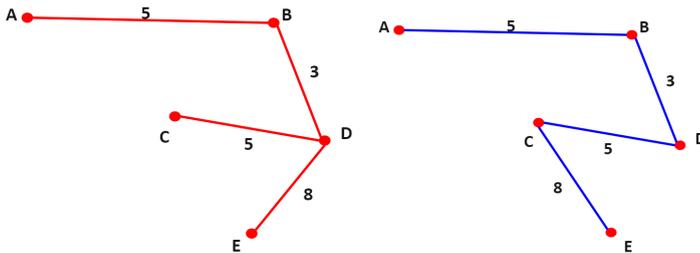
Question 1

- (a) Total length = 21 m
- (b) In a different context the weight could represent cost, time, etc. and could be affected by economics, availability (cost) or traffic, skill level (time).



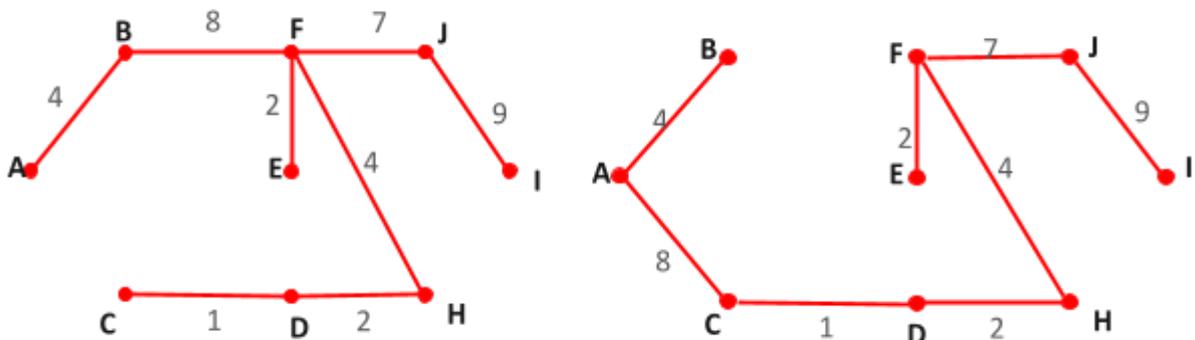
Question 2

- (a) Weight = 21 m



- (b) Prim's starts with a node. Kruskal's starts with an edge. Prim can start with any vertex while Kruskal must start with the edge of least weight. The tree grows in a connected fashion when using Prim while Kruskal's can grow separately.

Question 3

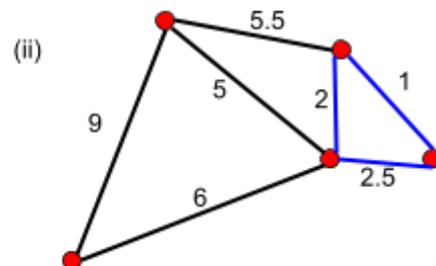
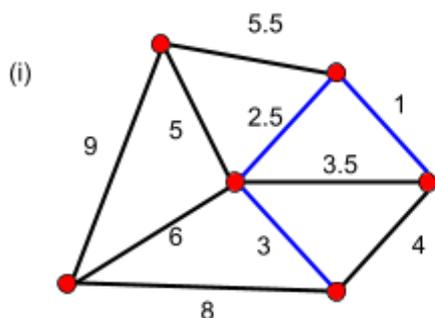


Weight = 37

Question 4

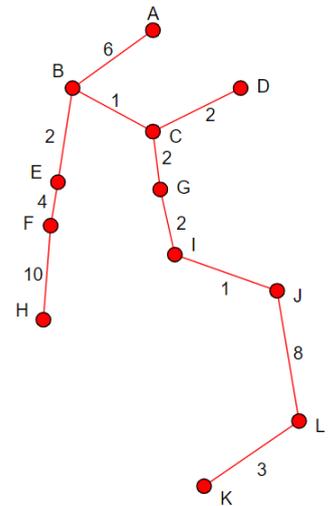
- (a) Kruskal's algorithm should be used for sparse graphs (low number of edges) and Prim's algorithm should be used for dense graphs (high number of edges).

- (b)



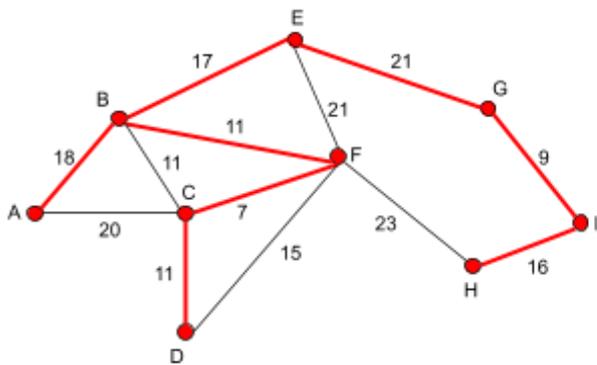
Question 5

- (a) A greedy algorithm is an algorithm that builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.
- (b) Weight = 41 km, Cost = €14,350
Note: This is one of many valid solutions.



Question 7

- (a) Kruskal's algorithm begins by selecting the edge of least weight.
- (b) Total length = 110 m



or

