Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2018

Applied Mathematics

Chief Examiner's Report

Contents

1. Introduction

This Chief Examiner's report provides a review of the performance of candidates in the 2018 Leaving Certificate Applied Mathematics examinations. It provides an analysis of candidate achievement at both Higher level and Ordinary level. Tables of relevant statistical information are given in the appendix. This report should be read in conjunction with the examination papers, the published marking schemes and the syllabus for this subject. The examination papers and marking schemes are available on the State Examination Commission's website www.examinations.ie and the syllabus is available at www.curriculumonline.ie

The current Applied Mathematics syllabus has been in existence for over 40 years. In common with others of this vintage, it is a brief list of the content of the course and does not deal with the aims or rationale of the subject, nor does it contain any objectives for the course. The content is essentially a set of topics from the domain of physics known as mechanics. The syllabus recognises that the subject is dependent on competence in mathematics in order to successfully access the course. It states that "Knowledge of the relevant parts of the mathematics course is assumed."

Notwithstanding the absence of explicitly stated aims, these can be inferred from the topics listed in the syllabus and the established style of the examination questions. Leaving Certificate Applied Mathematics aims to foster problem-solving skills. The course develops the learner's capacity to use mathematics to solve mechanics problems. The learner analyses a given problem, formulates a mathematical representation of the problem, solves the mathematical problem and interprets and verifies the results. By following this course students should learn to appreciate the extent to which mathematics is relevant and useful in the solutions of a range of problems.

The syllabus is differentiated into Higher and Ordinary level in terms of range of material, depth of treatment and assumed familiarity with relevant mathematical procedures. All Ordinary level topics are on the Higher-level syllabus, whereas some additional material has been designated for Higher level only. Higher level requires a greater depth of understanding of concepts, processes and principles and a greater degree of proficiency in skills, as well as knowledge of topics on the Higher-Level Mathematics Syllabus (such as integration).

The current Leaving Certificate Higher level Mathematics syllabus, in place since 2015, has placed less emphasis on the mathematical skills of algebraic and trigonometric manipulation that candidates would have been familiar with prior to that syllabus change. In addition, the range of methods of integration has been reduced. It is therefore to be expected that the proficiency of the majority of candidates in these areas would not be at the same level as had been the case prior to the change in the mathematics syllabus, unless more time in the Applied Mathematics classroom were to be devoted to these skills.

2. What does the examination test?

Although the syllabus does not include explicitly stated objectives, any mathematical problem-solving course has an implied set of objectives that form the commonly understood cornerstone of this activity. For the purposes of designing the examination each year, the objectives of Leaving Certificate Applied Mathematics are to develop applied mathematical problem-solving skills so that students will, in the context of the course content, be able to:

- Analyse a problem: demonstrate knowledge and understanding of described physical processes and be able to interpret how the processes will evolve over time and in the context of any constraints

- Translate the problem into mathematics: create or choose a suitable mathematical model, and then formulate the question as a mathematical problem within the model.

- Compute a solution: use mathematical techniques to solve the mathematical problem and interpret that solution in the context that gave rise to the problem.

The examination directly assesses the candidates' capacity to do this, by presenting them with such problems, contextualised within the content domain specified.

At both levels, the examination consists of a single two-and-a-half-hour written paper. At Ordinary Level, there are nine questions, of which the candidate must answer any six. At Higher Level, there are ten questions, and here too the candidate must answer any six. At each level, a question will generally require knowledge of only one or at most two topics from the syllabus list. Therefore, given the element of choice in the examination, some candidates do not cover all of the topics listed when preparing for the paper. This strategy reduces candidates' choice of questions, as well as limiting the understanding of the subject.

3. How well did the 2018 candidates achieve the objectives of the course, and how do we know?

The 2018 candidates' engagement with and achievement of the objectives of the syllabus are considered in turn below. Commentary is based on an analysis of candidates' responses to problems set in specific questions (or sections of questions) on the written paper at both levels. The commentary is based on feedback provided by examiners and the Chief Advising Examiner in their final reports and collated by the Chief Examiner.

3.1 Analyse a problem: demonstrate knowledge and understanding of described physical processes and be able to interpret how the processes will evolve over time and in the context of any constraints

Candidates at both levels were required to demonstrate knowledge and understanding of described physical processes and show capacity to interpret how the process will evolve over time and in the context of any constraints or conditions applied to the process in the question.

The objective of demonstrating knowledge and understanding of described physical processes was achieved well at Higher level, as evidenced by the fact that most candidates had little difficulty in creating appropriate mathematical models for these processes.

Question 1(a) at Higher level described a parcel about to slide on a rough floor in a moving van. The solution to the problem involved elements from two separate topics that would not normally have been associated in a single context. Most candidates showed good levels of knowledge and understanding of the described process. They were able to relate cause and effect of forces on the parcel. Most candidates approached the question logically and were able to apply their knowledge in a concise manner and proceeded to create relevant equations to solve. Most candidates arrived at a mathematical result which they then interpreted and stated their conclusion relating to the information given in the original question.

In question 3(a)(ii), candidates had a choice of two methods of solution: equating the vertical displacements or equating the sum of the horizontal displacements to 100. Both approaches were common.

Likewise, in question 8(b), the majority of candidates who attempted this question showed a clear understanding of the described situation involving a disk with holes in it, and how this situation would lead to the disc oscillating when allowed to turn freely about a fixed point. However, in question 6(b)(ii) on circular motion, a majority of the candidates who attempted this question were not able to demonstrate the level of understanding of the situation that was required in order to identify the occasions when the tension in the string would be at its maximum or at its minimum.

At Ordinary level, in question 5 (a)(ii), alternative approaches to solving a problem were noted – while the majority of candidates found and subtracted the combined kinetic energy before and after the collision, some others found the change in kinetic energy for each sphere and then combined the answers, leading to a similar conclusion.

In a small number of instances, candidates misinterpreted the description given in the question. At Higher level, in question 2(a) on relative velocity, a number of candidates misinterpreted the direction that the aircraft travelled in still air. Similarly, in question 6(b) on circular motion, a minority of candidates did not take the tension in the string into account.

At Ordinary level, in question 4(b), some candidates did not include the reaction force on the diagram and some showed one of the weights as being perpendicular to the inclined plane rather than vertical.

3.2 Translate the problem into mathematics: create or choose a suitable mathematical model, and then formulate the question as a mathematical problem within the model.

As a consequence of the well-established form of the question papers and the length of time that the syllabus has been in place, many of the questions involve a comparatively familiar type of problem that candidates have prepared for and understand. Perhaps unsurprisingly in this context, the great majority of candidates at Higher level demonstrated in their responses that they were adept at creating or choosing a suitable mathematical model for the problem, and then formulating the question as a mathematical problem within the model.

At Higher level, in question 5(a) and 5(b) candidates used appropriate equations to describe the interactions of particles during a collision. Similarly, in question 4, the formulating of the equations of motion of connected particles was of a very high level of competence in almost all cases. However, in question 1(b) on linear motion with two cars travelling for different periods of time, some candidates did not distinguish between the two times concerned (for example, they used t to represent both quantities instead of t_1 and t_2), leading to inappropriate cancellations when solving.

At Ordinary level, candidates had little difficulty in writing appropriate equations to model the problems described in the questions. However, in question 5(b)(ii), which involved a ball dropping and rebounding, the equations produced by some candidates did not properly address the stated conditions. Their equations did not match the information provided on the different heights of the ball and on how gravity was speeding up the ball before the rebound and retarding it afterwards.

3.3 Compute a solution: use mathematical techniques to solve the mathematical problem and interpret that solution in the context that gave rise to the problem.

At Higher level, algebraic and arithmetical skills were required throughout the examination to solve mathematical models of real-world situations. Trigonometric methods were required in questions 2, 3, 4, 5, 7 and 9. Calculus was required in questions 2 and 10.

The higher achieving candidates demonstrated very good ability in mathematical problem solving. However, for a significant number of candidates, poor algebraic and trigonometric skills, and difficulties with integration, were key factors in preventing higher achievement.

Those candidates who achieve at least moderately high grades in Applied Mathematics generally show competence in each of the implied course objectives. The discriminating factor among these candidates tends to be how well they are able to use appropriate mathematical techniques in finding their solutions. The highest achieving candidates were those who were concise and accurate in applying their mathematical skills to the equations in the models they had selected for the problems. Generally, candidates who achieve lower grades struggle to perform well at this objective.

As one examiner stated, "Applied Mathematics concepts are in general well applied. The key issues in determining the outcome for the candidate are around understanding of and ability to apply algebra, trigonometry and calculus. Greater effort needs to be put into perfecting these skills to ensure higher achievement in the applied mathematics course."

At Higher level, in question 2, which was about relative velocity, there was an over-reliance on a vector approach. Few candidates showed an understanding of how to use an appropriate diagram, the concept of maximisation and the sine rule to get the required answers, which would have been a more efficient approach.

Also, in part (b) of question 10, where candidates were required to find the integral of a function, many candidates had difficulty in separating the variables of the first order integral, and they consequently failed to carry out the integration with respect to x correctly.

At Ordinary level, algebraic and arithmetical skills were required throughout the examination to solve problems within mathematical models of real-world situations. Trigonometric methods were required in questions 2, 3, 4, 7 and 8. Similar to Higher level, the highest achieving candidates were those who were concise and accurate in applying their mathematical skills to the equations in the models they had selected for the problems.

4. What can current and future students and their teachers learn from this?

- Candidates appear to be very confident in certain areas of the course, typically the earlier areas involving linear acceleration, relative velocity, projectiles and connected particles. Candidates demonstrated a good knowledge of the techniques involved and it is apparent that they have spent much time practising and perfecting these techniques. However, many students appear to have learnt the techniques by repeated practice of similar questions and are less able to apply the same techniques in a different context. For example, candidates were well able to resolve velocities into components; however, they then were unable to apply the same skill as successfully to resolve forces into components. Candidates should spend more time on the understanding of the concepts involved and considering how they might be broadly applied, rather than just concentrating on a particular application of the technique involved. They need to understand the concepts underlying these techniques in order to apply them generally and be better equipped to deal with questions that are not standard.

- From the distribution of the questions attempted by groups of candidates in particular centres, it is possible to conclude that candidates are focussing on specific areas of the curriculum to the exclusion of others. It would be advisable to allocate enough time to cover each section of the curriculum. This would give candidates a wider choice of question in the examination and improve their overall chances of success, as well as giving them a broader understanding of the subject. That is, complete coverage of the syllabus is recommended. Confining syllabus coverage to six or seven topics limits choice and may also present difficulties if parts of different topics are examined in the same question.

- More time needs to be spent on general mathematics skills, including handling fractions, algebraic manipulation, trigonometric identities, rules of logs, separation of variables in integration problems, and on the skills of integration.

- Answering six questions in the time available, especially at Higher level, requires prudent time management and, in particular, the self-discipline to move on to the next question when necessary. Time management and sticking to the time one has allocated to each question can be a significant factor in the achievement of candidates.

- Candidates are advised to practise force diagrams and to pay attention to the question of whether friction does or does not apply.

- Draw large, clear, labelled diagrams where appropriate. This is particularly important, as in most cases equations must be formed using the forces in the diagram and the direction or sense of the force is significant.

- It is advisable to start each question on a left-hand page as errors in transposition across page turns are common.

Appendix: statistics & trends

Participation trends

Year	*Applied Mathematics* candidature	Total Leaving Certificate candidature*	*Applied Mathematics* as % of total
2014	1706	52772	3.2
2015	1919	54025	3.6
2016	2090	55047	3.8
2017	1958	55707	3.5
2018	1955	54440	3.6

*Total Leaving Certificate candidature excludes Leaving Certificate Applied candidates.

Table 1: participation in Leaving Certificate Applied Mathematics, 2014 to 2018

Year	Total *Applied Mathematics* candidature	Number at Ordinary level	Number at Higher level	% Ordinary level	% Higher level
2014	1706	137	1569	8.0	92.0
2015	1919	190	1729	9.9	90.1
2016	2089	172	1917	8.2	91.8
2017	1958	100	1858	5.1	94.9
2018	1955	128	1827	6.5	93.5

Table 2: number and percentage of candidates at each level, 2014 to 2018

Year	Total Higher level	Female Candidates	Male Candidates	Female as % of total	Male as % of total
2014	1569	412	1157	26.3	73.7
2015	1729	432	1297	25.0	75.0
2016	1917	446	1471	23.3	76.7
2017	1869	486	1383	26.0	74.0
2018	1826	496	1330	27.2	72.8

Table 3: gender composition of Higher level subject cohort, 2014 to 2018

Year	Total Ordinary level	Female Candidates	Male Candidates	Female as % of total	Male as % of total
2014	137	28	109	20.4	79.6
2015	190	44	146	23.2	76.8
2016	172	29	143	16.9	83.1
2017	100	24	76	24.0	76.0
2018	128	29	99	22.7	77.3

Table 4: gender composition of Ordinary level subject cohort, 2014 to 2018

Overall performance of candidates

The grading scale for Leaving Certificate examinations changed in 2017. Direct comparison with all aspects of the grade distributions from previous years is not possible. For this reason, only comparative data from 2017 are presented in the tables below.

Year	Total	1	2	3	4	5	6	7	8
2017		14.4	23.1	20.6	15.3	11.5	7.6	4.3	3.2
2018		15.1	22.1	20.3	15.1	11.8	8.3	3.6	3.7

Table 5 Percentage of candidates awarded each grade in *Applied Mathematics* at Higher level, 2017 and 2018

Year	Total	1	2	3	4	5	6	7	8
2017		9.7	20.4	24.7	19.1	13.2	8.0	3.9	1.0
2018		10.3	21.2	23.8	14.3	12.9	9.3	3.8	3.4

Table 6 Percentage of female candidates awarded each grade in Applied Mathematics, at Higher level, 2017 and 2018

Year	Total	1	2	3	4	5	6	7	8
2017		16.1	24.0	19.2	14.0	10.9	7.4	4.5	3.9
2018		16.8	22.4	19.0	15.4	11.4	8.0	3.5	3.5

Table 7 Percentage of male candidates awarded each grade in Applied Mathematics, at Higher level, 2017 and 2018

Year	Total	1	2	3	4	5	6	7	8
2017		15	13	24	16	8	9	3	12
2018		20.3	22.7	14.1	19.5	8.6	7.8	2.3	4.7

Table 8 Percentage of candidates awarded each grade in Applied Mathematics, at Ordinary level, 2017 and 2018

Year	Total	1	2	3	4	5	6	7	8
2017		8.3	4.2	29.2	12.5	12.5	16.7	0.0	16.7
2018		27.6	17.2	20.7	10.3	3.4	13.8	3.4	3.4

Table 9 Percentage of female candidates awarded each grade in Applied Mathematics, at Ordinary level, 2017 and 2018

Year	Total	1	2	3	4	5	6	7	8
2017		17.1	15.8	22.4	17.1	6.6	6.6	3.9	10.5
2018		18.2	24.2	12.1	22.2	10.1	6.1	2.0	5.1

Table 10 Percentage of male candidates awarded each grade in Applied Mathematics, at Ordinary level, 2017 and 2018

Engagement with and performance on individual questions

The data in tables 11 and 12 are based on an analysis of the total number of candidates.

Performance			Popularity		
Question	Average Mark (%)	Rank Order	Response Rate (%)	Rank Order	Topic of Question
1	70·2	5	86·6	4	Linear Motion
2	65·3	7	75·0	6	Relative Velocity
3	70·9	4	92·4	2	Projectiles
4	76·5	1	93·7	1	Connected Particles
5	72·2	2	91·1	3	Collisions
6	54·4	10	9·3	10	Circular Motion & SHM
7	55·3	9	10·5	9	Statics
8	69·2	6	27·2	7	Rigid Body Motion
9	65·2	8	20·5	8	Hydrostatics
10	72·0	3	85·6	5	Differential Equations

Table 11: attempt rate and average mark for each question, Higher Level Applied Mathematics,

Performance			Popularity		
Question	Average Mark (%)	Rank Order	Response Rate (%)	Rank Order	Topic of Question
1	86·4	1	98·4	1	Linear Motion
2	69·6	5	89·8	5	Relative Velocity
3	71·8	3	95·3	2	Projectiles
4	84·2	2	93·8	3	Connected Particles
5	66·6	6	92·2	4	Collisions
6	71·2	4	38·3	7	Centre of Gravity
7	31·2	9	7·0	9	Statics
8	64·8	7	39·8	6	Circular Motion
9	58·4	8	28·1	8	Hydrostatics

Table 12: attempt rate and average mark for each question, Ordinary Level Applied Mathematics,