PDST

L. C. Applied Maths: Student Learning Outcomes

Strand 1: Mathematical Modelling

Students should be able to:

Students learn about	Students should be able to
The problem-solving cycle	1.1 describe a systematic process for solving problems and making decisions
Formulating problems	1.2 Research the background to a problem to analyse factors or variables that affect the situation 1.3 Determine information relevant to the problem 1.4 Decompose problems into manageable parts 1.5 Determine what assumptions are necessary to simplify the problem situation

Students learn about	Students should be able to
Translating problems into mathematics	1.6 Use abstraction to describe systems and to explain the relationship between wholes and parts 1.7 Abstract the knowledge needed to build a mathematical model 1.8 Translate the information given in the problem together with the assumptions into a mathematical model that can be solved
Computing Solutions	1.9 Compute a solution using appropriate mathematics 1.10 Create a mathematical model that can be interpreted by a computer

Students learn about	Students should be able to
Computing Solutions	1.11 Use computational technology to solve problems 1.12 Solve the mathematical problem stated in the model 1.13 Analyse and perform operations in the model 1.14 interpret the mathematical solution in terms of the original situation
Evaluating solutions	1.15 Refine a model and use it to predict a better solution to the problem; iterate the process 1.16 Communicate solution processes in a written report

Strand 2: Mathematical Modelling with networks and graphs

Students should be able to:

Students learn about	Students should be able to
Networks and their associated terminology	2.1 Represent real-world situations in the form of a network 2.2 Use and apply the following network terminology: vertex / node, edge/arc, weight, path, cycle 2.3 Distinguish between connected and disconnected graphs, and between directed and undirected graphs 2.4 Represent a graph using an adjacency matrix, and reconstruct a graph from its adjacency matrix
Matrices, matrix algebra and adjacency	2.5 Perform multiplication of square matrices by hand, with the help of a computer for larger matrices 2.6 Interpret the product of adjacency matrices 2.7 Translate between multiple representations of mathematical ideas

Students learn about	Students should be able to
Minimum spanning trees applied to problems involving optimising networks and algorithms associated with finding these (Kruskal, Prim)	2.8 Demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree in appropriate contexts 2.9 Use appropriate algorithms to find minimum spanning trees
Dynamic Programming and shortest paths as applied to multi-stage authentic problems such as: - stock control - routing problems - allocation of resources - equipment replacement and maintenance	2.10 Use and apply dynamic programming terminology, such as stage, state, optimal policy 2.11 Apply Bellman's Principle of Optimality to find the shortest paths in a weighted directed acyclic network, and to be able to formulate the process algebraically 2.12 Apply Dijkstra's algorithm to find the shortest paths in a weighted undirected and directed network 2.13 Evaluate different techniques for solving shortest-route problems

Students learn about	Students should be able to
Algorithms: - Dijkstra - Bellman - Kruskal, Prim	2.14 Use algorithms to solve problems **2.15 Distinguish between those algorithms which are greedy and those which use dynamic programming** **2.16 Justify the use of algorithms in terms of correctness and their ability to yield an optimal solution**
Analysis of project scheduling networks Project scheduling and its associated terminology	2.17 Apply network concepts to project scheduling 2.18 Apply the concepts of critical path, early times, late times and floats to project scheduling

Strand 3: Mathematical Modelling the physical world; kinematics and dynamics

Students should be able to:

Students learn about	Students should be able to
Kinematics: Particle motion in one direction. - Common idealisations used to simplify authentic situations involving motion in 1D. - Five basic concepts for describing the motion of a particle in 1D; position, displacement, velocity, acceleration and time. - Graphical Representation and interpretation: displacement -time graphs, velocity-time graphs - The kinematics formulae under constant acceleration • $v = u + at$ • $s = ut + \frac{1}{2}at^2$ • $v^2 = u^2 + 2as$	3.1 Describe the motion of a particle in 1D [In a straight line] using words, diagrams, numbers, graphs and equations
Displacement as a change of position Velocity as the rate of change of displacement Acceleration as the rate of change of velocity Differentiation • The chain rule as a technique for differentiating composite functions The fundamental theorem of calculus The definite integral as the area under the curve (Riemann integral)	3.2 Interpret velocity and acceleration as derivatives 3.3 Transform the function describing one quantity (displacement, velocity, acceleration) into functions describing the other two quantities using algebra and/or calculus 3.4 Solve kinematics problems involving particle motion in one dimension 3.5 Derive the kinematic formulae of motion using calculus
Particle motion in 2D Elementary vector Algebra • addition – adding 2 successive displacements or velocity vectors • scalar multiplication • dot-product	3.6 Represent displacement as a vector 3.7 Apply and interpret vector algebra 3.8 Represent vectors in terms of components along unit vectors in 2 fixed orthogonal directions and in polar form 3.9 Calculate and interpret the dot product of vectors

Students learn about	Students should be able to
Elementary vector calculus Projectile Motion; time of flight, maximum height, maximum range, horizontal planes	3.10 transform the function describing one quantity (displacement, velocity, acceleration represented as vectors) into functions describing the other two quantities using algebra and/or calculus 3.11 represent and apply Newton's laws in vector form 3.12 solve constant acceleration projectile motion problems involving displacement, velocity and time
Forces acting on a particle Mass Types of Forces; • Applied, Normal Reaction, Frictional, Resistant, Tension • Gravitational • Resultant Vectors as representations of quantitative data The concept of momentum Newton's Laws of motion Impulse Conservation of momentum for a two-particle system in one and two dimensions Newton's experimental laws for collisions	3.13 define a force as a measurable quantity 3.14 draw free-body force diagrams for a particle on a smooth rigid fixed horizontal or inclined plane 3.15 find the resultant force along a plane or inclined plane 3.16 resolve forces on rough and smooth surfaces 3.17 solve dynamic problems involving the motion of a particle under a constant resultant force 3.18 solve dynamic problems involving particles that collide directly **and obliquely** 3.19 solve dynamic problems involving connected masses 3.20 describe the motion of a particle on a smooth/ rough, horizontal or inclined plane under a constant resultant force 3.21 solve dynamic problems on rough and smooth surfaces

Students learn about	Students should be able to
Elastic and inelastic collisions Coefficient of restitution. Connected masses Rough and smooth surfaces Coefficient of friction Drag: Liquid, aerodynamic where $F \propto v^n$ Integration • by parts derived from the product rule • by substitution derived from the chain rule	3.22 solve dynamic problems involving resistive forces that are proportional to v^n $n \in$ **R**
Work, energy, conservative forces, conservation of energy and momentum A work-energy model for analysing particle motion The principle of conservation of energy Work done by a force • Gravitational force under the constant acceleration approximation • Variable conservative force • Force exerted on a particle by a linear stretched or compressed elastic spring and stretched strings	3.23 define work done 3.24 describe gravitational potential energy and how it relates to work done 3.25 describe kinetic energy and how it relates to work done 3.26 solve dynamic problems involving work done by a constant force 3.27 solve dynamic problems involving the conservation of energy for variable conservative forces
Circular Motion of a particle	3.28 solve problems involving the dynamics of a particle moving in a horizontal or vertical circle
Dimensional analysis	3.29 evaluate and articulate whether an answer is reasonable by analysing the dimensions

Strand 4: Mathematical Modelling a changing world

Students should be able to:

Students learn about	Students should be able to
Recurrence relations	4.1 compute the first and higher differences of a given sequence of numbers
Real-world phenomena involving incremental change such as - Malthusian growth, restricted growth - interest/loan payment - supply and demand - spread of diseases such as measles	4.2 identify real-world situations which can be suitably modelled by difference equations 4.3 derive difference equations for real-world phenomena involving incremental change 4.4 analyse, interpret and solve difference equations in context

Students learn about	Students should be able to
Solving homogeneous and inhomogeneous difference equations numerically, graphically and with the aid of digital technology	4.5 solve linear and non-linear difference equations
Analysing real-world phenomena involving continuous change Predictive nature of differential equations	**4.6 identify real-world situations which can be suitably modelled by differential equations** **4.7 derive and interpret in context differential equations for real-world phenomena involving continuous change**

Students learn about	Students should be able to
Techniques for solving differential equations • separation of variables • numerical solutions for digital implementation • graphical methods that provide qualitative information	**4.8 solve differential equations** **• first order separable** **• second order which can be reduced to first order** **4.9 interpret the solution of differential equations in context**