Problem Solving Evening Workshops

Participant Booklet

PDST Professional Development Service for Teachers

Contents

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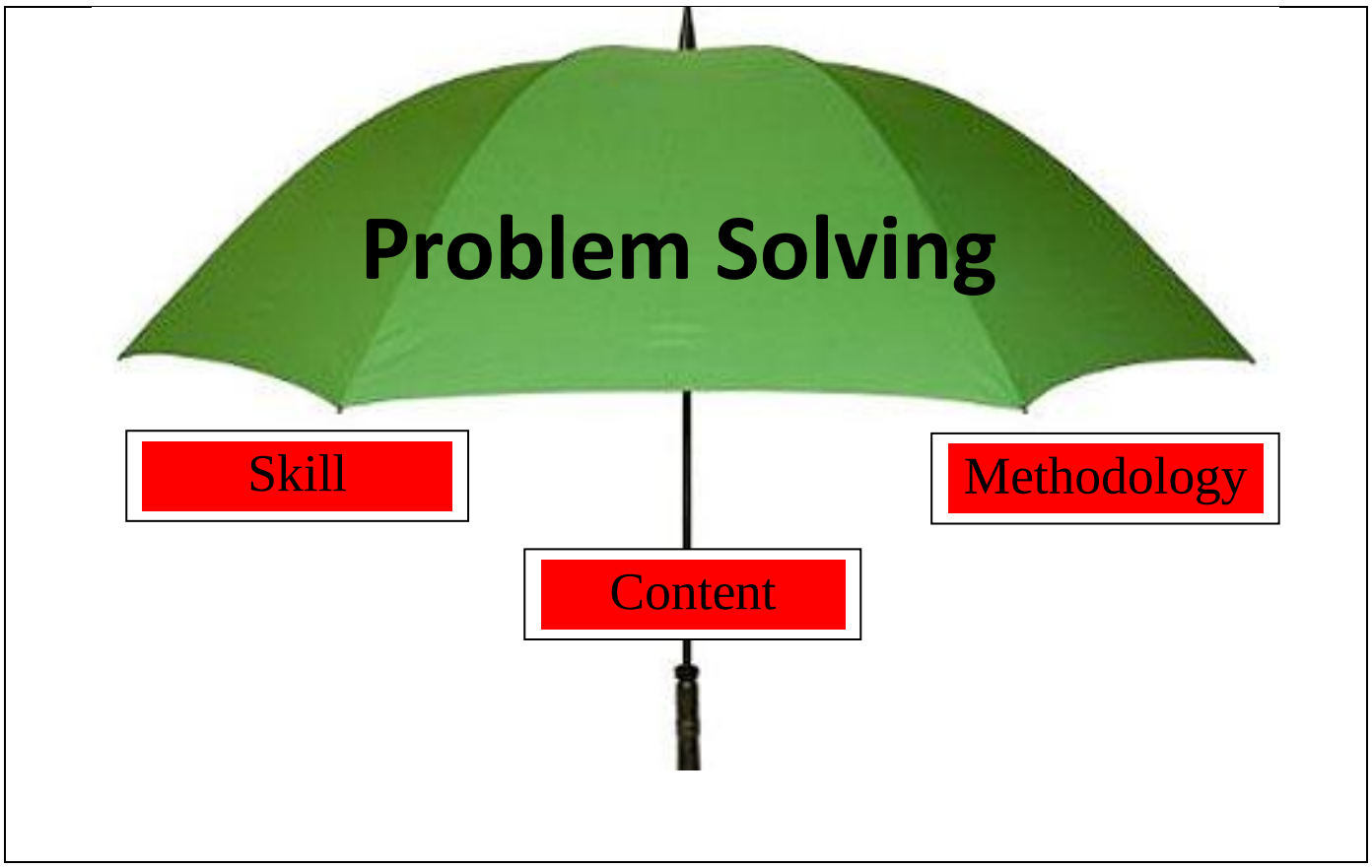
Introduction



Problem Solving can be viewed as an overarching, multi-faceted approach that incorporates skill, content and methodology. Problem solving can be developed as a skill with reference to the key skills of the primary school mathematics curriculum. In relation to content, problem solving is evident in terms of problems that are encountered in the mathematics curriculum, in addition to problems in other areas of the curriculum. Relating to methodology, a problem solving approach to teaching can also be employed in mathematics lessons (methodology).

Strategies for Supporting and Developing Mathematical Thinking – Instructional Framework

Eliciting	Supporting	Extending
Facilitates pupils' responding Elicits many solution methods for one problem from the entire class *e.g. "Who did it another way?; did anyone do it differently?; did someone do it in a different way to X?; is there another way of doing it?"* Waits for pupils' descriptions of solution methods and encourages elaboration Creates a safe environment for mathematical thinking *e.g. all efforts are valued and errors are used as learning points* Promotes collaborative problem solving ***Orchestrates classroom discussions*** Uses pupils explanations for lesson's content Identifies ideas and methods that need to be shared publicly *e.g. "John could you share your method with all of us; Mary has an interesting idea which I think would be useful for us to hear."*	***Supports describer's thinking*** Reminds pupils of conceptually similar problem situations Directs group help for an individual student through collective group responsibility Assists individual pupils in clarifying their own solution methods ***Supports listeners' thinking*** Provides teacher-led instant replays *e.g. "Harry suggests that ...; So what you did was ...; So you think that ...".* Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method *e.g. "I have an idea ...; How about ...?; Would it work if we ...?; Could we ...?".* ***Supports describer's and listeners' thinking*** Records representation of each solution method on the board Asks a different student to explain a peer's method *e.g. revoicing (see footnote on page 8)*	*Maintains high standards and expectations for all pupils* Asks all pupils to attempt to solve difficult problems and to try various solution methods ***Encourages mathematical reflection*** Facilitates development of mathematical skills as outlined in the PSMC for each class level *e.g. reasoning, hypothesising, justifying, etc.* Promotes use of learning logs by all pupils *e.g. see Appendix A for a sample learning log* ***Goes beyond initial solution methods*** Pushes individual pupils to try alternative solution methods for one problem situation Encourages pupils to critically analyse and evaluate solution methods *e.g. by asking themselves "are there other ways of solving this?; which is the most efficient way?; which way is easiest to understand and why?".* Encourages pupils to articulate, justify and refine mathematical thinking *Revoicing can also be used here* Uses pupils' responses, questions, and problems as core lesson including student-generated problems ***Cultivates love of challenge***

This is adapted from Fraivillig, Murphy and Fuson's (1999) Advancing Pupils' Mathematical Thinking (ACT) framework.

Classroom Culture

Creating and maintaining the correct classroom culture is a pre-requisite for developing and enhancing mathematical thinking. This requires the teacher to:

- cultivate a 'have a go' attitude where all contributions are valued;
- emphasise the importance of the process and experimenting with various methods;
- facilitate collaborative learning through whole-class, pair and group work;
- praise effort;
- encourage pupils to share their ideas and solutions with others;
- recognise that he/she is not the sole validator of knowledge in the mathematics lesson;
- ask probing questions (see Appendix B for a list of sample questions and sample teacher language);
- expect pupils to grapple with deep mathematical content;
- value understanding over 'quick-fix' answers; and
- use revoicing1 (reformulation of ideas) as a tool for clarifying and extending thinking.

In this type of classroom pupils are expected to:

- share ideas and solutions but also be willing to listen to those of others; and
- take responsibility for their own understanding but also that of others.

Sample List of Questions and Teacher Language

Eliciting: Questioning and Teacher Language

Who did it another way?

Did anyone do it differently?

Did someone do it in a different way than Mary?

Is there another way of doing it?

John, could you share your method with all of us?

Mary has an interesting idea which I think would be useful for us to hear.

We will need to tidy our material away soon. Can we find a way of saving our ideas in another way?

Can anybody now explain to me in your own words what a _____ means?

Can anyone think of a way we can record this using numbers?

Using the material you have chosen to work with, can you show me what a _____ looks like?

How do you know?

How will we check?

Explain how you did it.

Supporting: Questions and Teacher Language

How about...?

Would it work if we...?

Could we...?

Harry suggests that...

So what you did was...

So you think that...

I have an idea...

Show me _____ in as many ways as possible.

Discuss with your partner what you need to do to solve the problem and estimate what the answer might be.

Mary has an interesting idea which I think would be useful for us to hear.

How could we draw a representation of this?

What materials could we use to represent this?

Extending: Questions and Teacher Language

Are there other ways of solving this?

Which is the most efficient way?

How could we do this in a more efficient way?

Which method is easiest to understand and why?

John, can you explain what Mary said in your own words?

Mary, can you repeat what John said using your own words?

Can you revoice what Mary said?

Check if this works for other examples.

How can we record the findings using symbols?

Record these using mathematical symbols.

Draw pictures to show how you worked out your answers.

Explain how you got your answer.

Explain what we have discovered.

Write about what you have learned in your learning log.

What did you find difficult about solving this problem?

How did your answer compare to your estimate?

What patterns did you notice?

Did you spot any patterns?

Could you suggest an algorithm for working out an equivalent fraction?

Test your algorithm to see if it works.

Test your hypothesis using other problems.

Rich Mathematics Problems

Rich mathematics problems are vitally important in mathematics lessons. Rich problems facilitate and promote mathematical thinking, in particular, communicating, reasoning, conjecturing and hypothesising. Two websites that are particularly useful for sourcing rich mathematical problems are www.nrich.maths.org and www.nzmaths.co.nz. The following is a rich maths problem based on weight and is taken from http://nrich.maths.org.

9 Weights (Stage: 3★)

You have been given 9 weights, one of which is slightly heavier than the rest. Can you work out which weight is heavier in just two weighings of the balance?

This is the core maths problem. Pupils can then be challenged to think more deeply about mathematics by extending this problem with the following:

Can you always work out which weight is heavier in just two weighings of the balance?

Could you always work out which is heavier in just two weighings if you had been given 10 weights?

You have now been given 27 weights, one of which is slightly heavier than the rest. Can you work out which weight is heavier in just three weighings of the balance?

Can you always work out which weight is heavier in just three weighings of the balance?
You now have 9 weights again, and know that one is a slightly different weight, but you don't know if it is heavier or lighter.
Can you work out which weight is different, and whether it is heavier or lighter, in just three weighings?
Can you explain how you would always be able to tell in just three weighings?

Mathematical Language

Mathematical language is a key tool in problem solving. Having good mathematical language skills helps pupils to be more successful problem solvers because they can more freely transfer skills from one concept to another. It also allows them to concentrate more on solving the problem, instead of mostly focusing on decoding the language.

Flashcard Activity

Take the four flashcards from your resource pack

- Place each of the four operations flashcards facing upwards on your desk
- Brainstorm in pairs or groups at least eight other ways of representing each operation, for example, Addition (+) can be represented as sum, plus, add etc.
- Record your answers on four separate post its and attach to each flashcard

  Addition (+)

  Subtraction (-)

  Multiplication (x)

  Division (÷)
- Could you use this strategy in your own classroom to discretely teach mathematical language?
- How could you extend this strategy to other strands and strand units? (For example in Measures what associated language could you teach around these terms?; Length?; Capacity?)

Mathematical Language

Teacher Test Activity

This is a teacher-designed task for teachers and by teachers! Can you complete the following activities taken from the glossary section of the Mathematics Teacher Guidelines: 1999 detailing the most commonly understood working definitions of some mathematical terms? Could you apply this idea to teaching and assessing mathematical language in your own classrooms?

Fill in the Answer

1. ---------- is a number denoting quantity but not order in a set.

2. ---------- the result of dividing one number by another number.

3. ---------- a letter or symbol that stands for a number.

4. ---------- tell at a glance, without counting, the number of items in a set.

Working Definitions

- **Ratio**

- **Diameter**

- **Commutative**

Capacity Crossword

1D A teaspoon holds about _ _ _ _ millilitres of water.

2D The classroom sink holds about twenty _ _ _ _ _ _ of water.

1A There are one thousand _ _ _ _ _ _ _ _ _ _ _ in a litre.

3D Over ninety per cent of the human body is _ _ _ _ _.

4D The _ _ _ _ _ _ is probably the heaviest user of water in the house. It uses 10 litres of water every time it is flushed.

5D The petrol tank of a medium sized car holds about _ _ _ _ _ litres of water.

2A One _ _ _ _ _ of a litre is the same as two hundred millilitres.

3A Put a quarter of a litre in a 1 Litre jug. Subtract 50 millilitres and 300 millilitres. The jug is now _ _ _ _ full

4A 0.1 of a litre is the same as one _ _ _ _ _ _ _ millilitres.

5A 0.01 of a litre is exactly _ _ _ millilitres.

6D A car tank that holds 60 litres of petrol used 75% of the contents on a journey. It had _ _ _ _ _ _ _ litres left at the end of the journey.

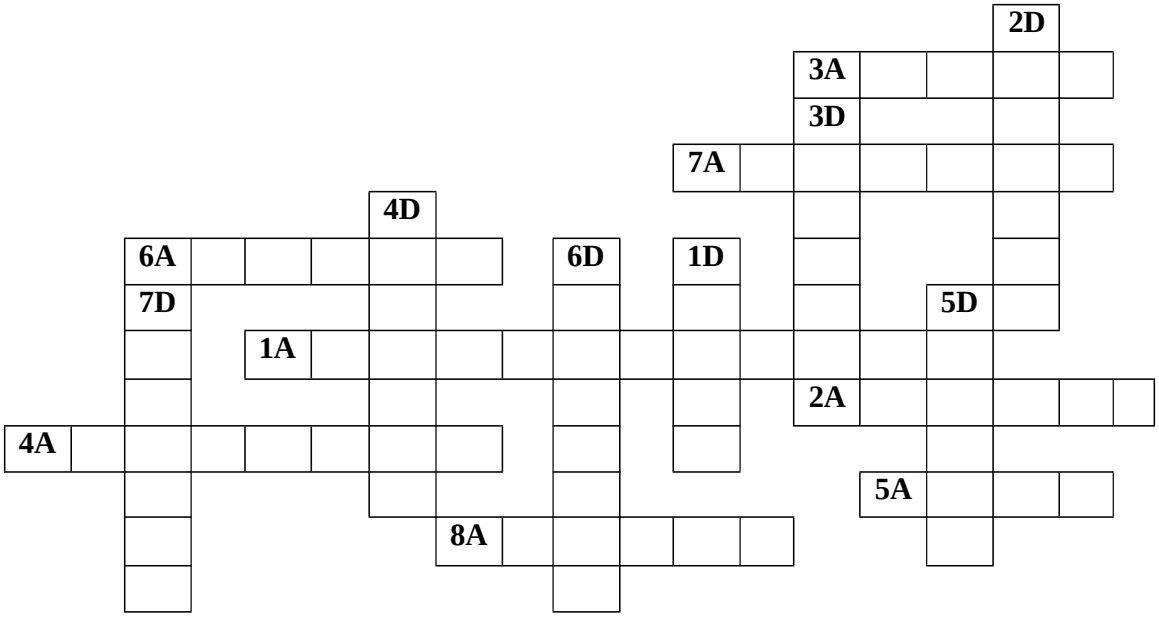
6A 75% of a litre is the same as seven hundred and _ _ _ _ _ millilitres.

7A 200 millilitres is _ _ _ _ _ _ per cent of a litre.

8A The average bath uses 80 litres of water. The average shower uses 35 litres. The ratio of bath water to shower water is sixteen as to _ _ _ _ _.

7D The bath uses more than _ _ _ _ _ _ the amount of water as the shower.

More crosswords can be found on the PPDS website at
http://ppds_en.educast.ie/index.php?option=com_content&task=view&id=262&Itemid=436


2D
3A
3D
7A
4D
6A
6D
1D
7D
5D
1A
2A
4A
5A
8A

Between Session Task

What will I do before the next session?
Notes for Feedback

Learning Log

Learning Log			
Name:		Class	
What strategy did I use to solve the problem?			
What would I do differently the next time?			
One thing I learned:			
My effort:			

Mathematics Journals

Journals dedicated to maths can be a very useful way for pupils to consolidate and communicate their learning. They can also aid mathematical thinking by promoting reflection, refinement and reasoning. Writing in mathematics can help pupils organise their understanding of mathematical ideas. In essence, writing is not simply a way for pupils to demonstrate what they know; it is a way to help them understand what they know.

Writing Prompts (Curriculum Content)

- The difference between a square and a rectangle is….
- How would you describe an acute angle?
- What are tangrams? Write as much as you can about them.
- Write all you know about the area of a triangle.
- Describe a litre, kilogramme, metre.
- Explain what horizontal lines.
- Write an explanation about the differences between area and perimeter.

Some writing prompts (Affective)

- How do you feel about maths since you started?
- I want to become better at maths because...
- My best day at maths was when...
- Write a story "If I were a centimetre tall"
- My aim in maths this year is to...
- I am improving at maths now because…
- One activity in maths I really enjoy is ….. because…

42 Ways to start a Mathematics Journal

1. I understand everything until I get to the part...
2. Another way I found to do this problem is
3. Today something new I learned today was...
4. I am still confused by...
5. I would like to know how...
6. Next time, I plan to...
7. I learned/understand...
8. I think is...
9. This problem is different/similar to because…
10. The (number) most important things I learned …
11. I feel confident /nervous about because…

12. I figured out this problem by….
13. A way I tried to solve this problem that didn't work out was … I think it didn't work out because...
14. The mathematical rule for the problem is…
15. I would explain this problem to a (different age group) by...
16. The thing I'm proud of myself for learning is...
17. The way I remember (how to) in my head is...
18. I'm wondering …
19. I know I'm right because …
20. It is easy/difficult for me to...
21. I find it harder/easier to...
22. I was surprised...
23. The possible patterns/perimeters/solutions to this problem are...
24. The dimensions are...
25. My partner/group and I...
26. If I were to describe to you...
27. The mathematical concept involved in this project...
28. I'm convinced that …
29. The information missing in this problem is...
30. The tricky part of the problem is…
31. The clue to look for …
32. For me, writing clues and solving them was...
33. The criterion I used...
34. The possibilities are...
35. It is more/least likely…
36. Chances are increased by...
37. The probability that…
38. A different way to solve this problem is…
39. I noticed/decided/accompolished/numbered/experimented/calculated/organised/counted/estimated/matched/measured/figured/realised/collected/compared/arranged…
40. Write 2 different ways to solve a problem (12x4 or 3+4)
41. Write the steps
42. Define a word, for example, what is a numerator?

Task Card

Dublin Zoo

Dublin Zoo has just received two new sheep for the family Farm part of the zoo. The zoo keeper wants to build an enclosure for the sheep. She decides that the enclosure must be square or rectangular with an area of 100 square metres.

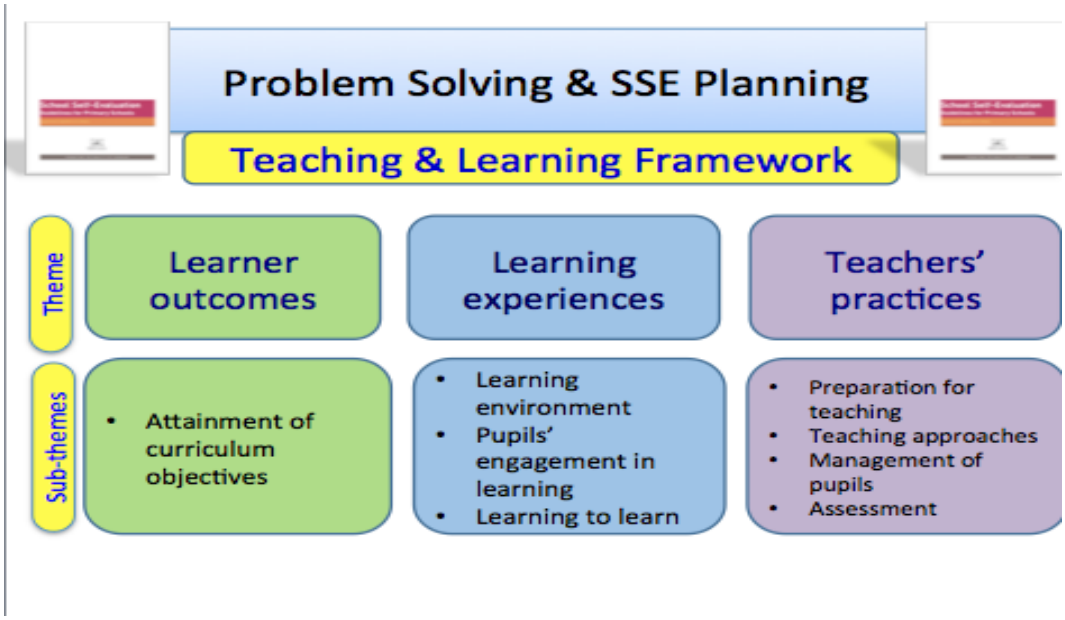
1. Which different configurations could she build?
2. How many metres of fencing will she need for each possible design?
3. Use your copy or some graph paper to draw all the possible rectangular or square designs.
4. Include a key to tell how much each unit on the grid paper equals.
5. Which fence would you recommend that the zoo keeper builds? Why?

- What questions do you have about the problem?
- What is the problem asking you to do?
- What do you know?
- What do you need to find out?

In groups of three, try to solve the problem.

Between Session Task

Notes for Feedback



The teaching and development of problem solving (PS) as a methodology, skill and content-based approach can be applied to each theme and subtheme of the teaching and learning framework from p.24/p.25 of the SSE Guidelines. This activity shows that PS spans all skills, methodologies, strands and strand units in the maths curriculum and is also a cross-curricular approach that can be applied to almost every aspect of the SSE guidelines – it is multi-faceted.

L.O. (Attainment of Curriculum Objectives)

This relates to pupils' learning and achievements in PS. Baseline data can be established in the form of readily available assessment data in schools, for example, analysis of standardised test data, diagnostic tests, teacher designed tasks and tests, etc. Knowledge, understanding, skills, attainment trends, etc. can be analysed so that learning targets for PS can be set.

L.E. (Learning Environment)

This relates to the quality of the physical setting in which the teaching and learning takes place. Examples of this include: How are ICT resources integrated / used to promote active learning in PS?; How are resources / concrete materials used in PS teaching & learning? Is pupils' PS work displayed and if so, in what ways?; How is a print rich PS environment in classrooms achieved / promoted?; etc.

L.E (Pupil Engagement)

This relates to pupils' learning experiences in terms of the range of approaches, etc. that are used. Examples of this include: How is active learning promoted throughout PS lessons?; Do pupils learn collaboratively or predominantly independently?; How are pupils challenged and supported

in PS?; Are pupils motivated to engage in PS?; How are intended PS learning outcomes shared with pupils?; etc.

L.E. (Learning to Learn)

This relates to how pupils' learning skills are developed. Examples include: How are pupils encouraged to communicate, engage, analyse and work with others in PS?; How are skills such as planning, presentation, communication and organisation in terms of PS modelled and taught?; How are pupils afforded the opportunity to develop, support and extend their learning through meaningful engagement with ICT PS resources?; How are pupils' taught how to monitor their own work and reflect on their PS approaches?; etc.

Teachers' Practices (Preparation for Teaching)

This relates to teachers' preparation in terms of PS planning, teaching & learning. Examples include: Do clear, specific differentiated learner outcomes relating to PS exist?; Is progress in PS specified in monthly records?; How are PS skills developed in other curricular areas?; How are AFL & AOL techniques and resources incorporated in planning for PS?; How is relevant information to inform and direct teaching through PS gathered?; etc.

Teachers' Practices (Teaching Approaches)

This relates to the effectiveness of the teaching approach used in all classrooms across the school. Examples include: Are a range of curriculum appropriate teaching methodologies used in PS?; How are pupils engaged in active learning, talk and discussion and collaborative learning in PS lessons?; How is content differentiated in PS?; How are AFL & AOL approaches used in PS teaching and learning?; How are positive pupil attitudes and dispositions developed in PS?; etc.

Teachers' Practices (Management of Pupils)

This relates to the nature of teacher–pupil interactions. Example include: How are PS activities structured?; What types of pupil-teacher interactions are promoted?; Are realistic and high expectations incorporated for all pupils in PS based instruction?; How is the pupil voice incorporated in PS?; etc.

Teachers' Practices (Assessment)

Sample questions include: How is assessment information used and analysed in PS?; How is pupil progress recorded and communicated in PS?; How is assessment incorporated where pupils are provided with constructive feedback on their learning?; Are these approaches in line with our assessment policy?; etc.

Where to next?