Prim's Algorithm

Prim's Algorithm was originally discovered in 1930 by Vojtech Jarnik and was then independently discovered by Robert Clay Prim in 1957.

1. To begin, pick any vertex/node (unless a predetermined one is indicated).
2. Find all the edges that connect the tree to new nodes, select the minimum and add it to the tree, ensuring to avoid cycles.
3. Keep repeating step 2 until we get a minimum spanning tree with all nodes connected and cycles avoided.

Note: It can be helpful to write a visited list to keep track of nodes that are already in the minimum spanning tree.

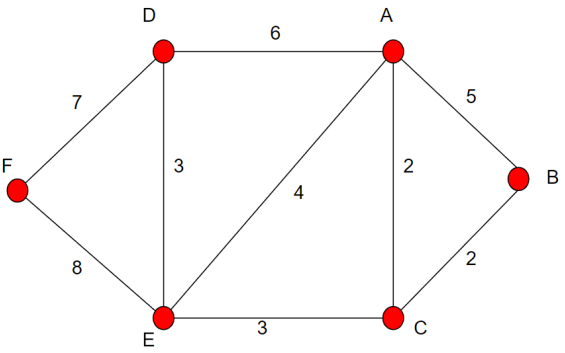
The **purpose** of Prim's Algorithm is to find a subset of the edges that forms a tree which includes every vertex where the total weight of all of the edges is a minimum.

Prim's algorithm is most suitable for dense graphs (large number of edges).

Additional Questions

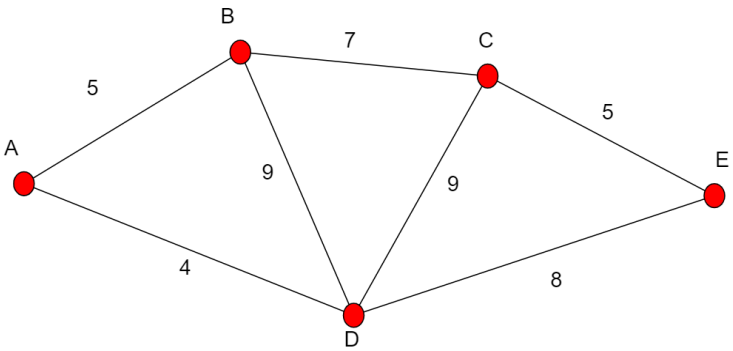
Question 1

Pauline is a gardener and has created a sprinkler system in the given diagram. Using Prim's algorithm, determine the network that will connect all of the sprinklers with the least amount of piping and determine the total length of piping needed. Each vertex represents a sprinkler and the weight of each edge represents the distance in *metres*.



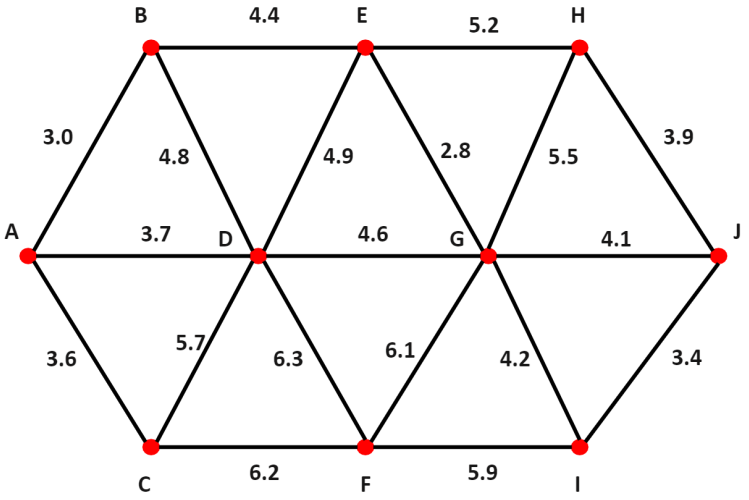
Question 2

The management of a Kerry campsite wants to connect each mobile home with running water in the easiest way possible. Each mobile home is represented by a letter and the weight on each edge represents the distance between the mobile homes in metres.
(i) Determine the Minimum Spanning Tree so that every mobile home is connected to running water using the least length of piping.
(ii) Calculate the total length of pipe needed.



Question 3

The network below shows the distances in metres between sprinklers on the ceiling of a room.



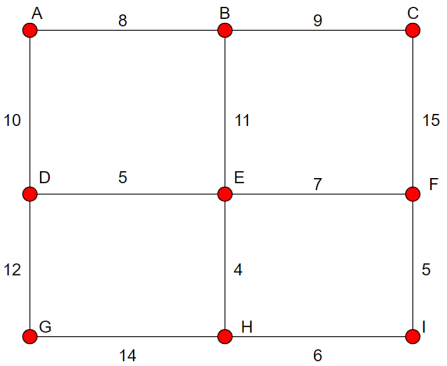
(a) A minimum spanning tree can be found using Prim's algorithm when starting at different points. State the final edge that would be added to complete the minimum spanning tree if the starting point were:
(i) A
(ii) F

(b) The same minimum spanning tree can be found using Kruskal's algorithm for the given network. Find the length of your minimum spanning tree showing the order in which you selected the edges.

Question 4

(a) Explain the terms:
(i) Tree
(ii) Spanning tree

(b) An electrician wants to wire sockets for a house using the least amount of wiring ensuring that they are all connected to each other. Draw the minimum spanning tree of the following graph and determine the total length of wiring needed. All distances are in metres.



Question 5

An energy supply company wishes to connect six villages in Connemara. The company will need to build a substation at one of the villages where the cost of building is the same at each village. The cost of connecting each village is outlined in the table below.

Cost of connection between each village						
	Cleggan	Moyard	Letterfrack	Tullycross	Garraun	Kylemore
Cleggan	-	€5,000	€8,000	€19,000	-	-
Moyard	€5,000	-	-	€8,000	€10,000	-
Letterfrack	€8,000	-	-	€11,000	-	€13,000
Tullycross	€19,000	€8,000	€11,000	-	€6,000	€25,000
Garraun	-	€10,000	-	€6,000	-	€21,000
Kylemore	-	-	€13,000	€25,000	€21,000	-

A '-' indicates no direct connection.

(a) Use Prim's algorithm to calculate the minimum cost energy supply network that would connect all 6 villages separate to the substation cost and show the minimum spanning tree.

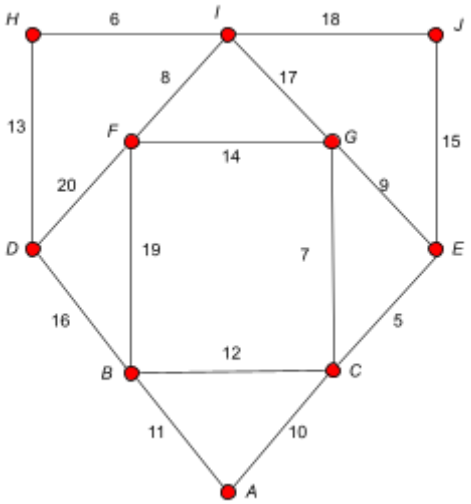
(b) A new minimum spanning tree is required which includes the links between Tullycross and Garraun as well as Garraun and Kylemore. Select and justify the most appropriate algorithm to solve this problem and determine the new cost.

Question 6

(a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices.

(ii) State the number of edges in a minimum spanning tree of a network with n vertices.

(b) The following network represents the footpaths in a park in Leitrim. There are 10 lamp posts at A, B, …, J. The numbers on each edge represent the distances, in metres, between lamp posts. The lamp posts are to be connected using cabling that follows the given footpaths.



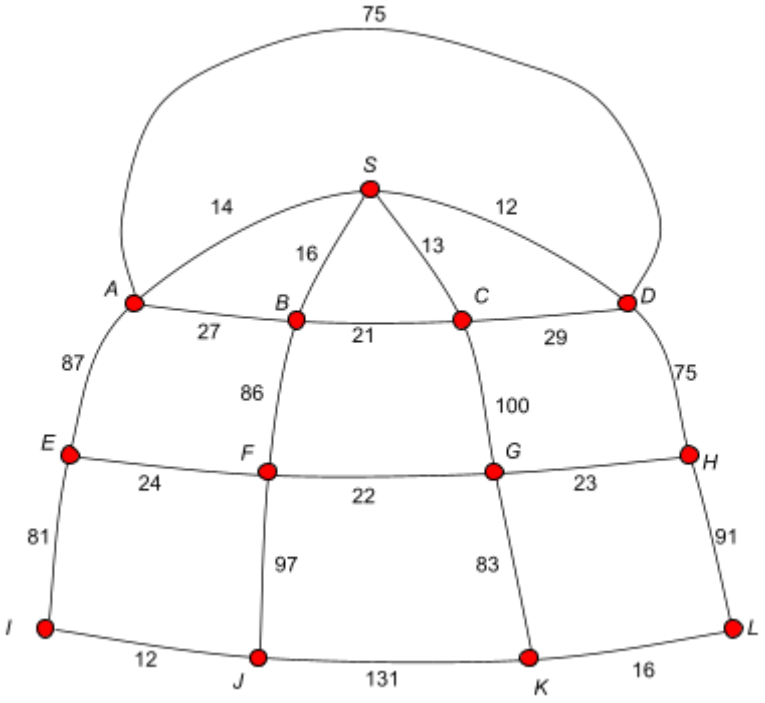
(i) Use Prim's algorithm to find the minimum length of cabling required to connect all of the lamp posts.

(ii) State the length of cabling required.

Question 7

The diagram shows the various tubing tracks at a tubing park in Laois. There is a shop at S. The manager of the tubing park intends to install a floodlighting system by placing a floodlight at each of the 12 points *A*, *B*, …, *L* and at the shop at *S*.

The number on each edge represents the distance, in metres, between two points.



Total of all edges = 1,135 *m*

The manager wishes to use the minimum amount of cabling, which must be laid along the tube tracks, to connect the 12 points A, B, …, L and the shop at S.

(a) Starting from the shop, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points.

(b) State the length of your minimum spanning tree.

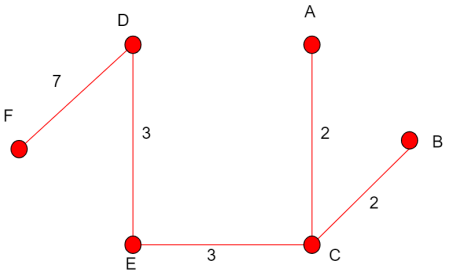
(c) Draw your minimum spanning tree.

(d) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and eighth edges that the manager added to their spanning tree.

Solutions:

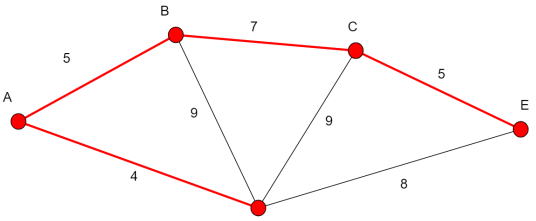
Question 1

Total distance = 17 *m*



Question 2

Length of pipe needed = 21 m



Question 3

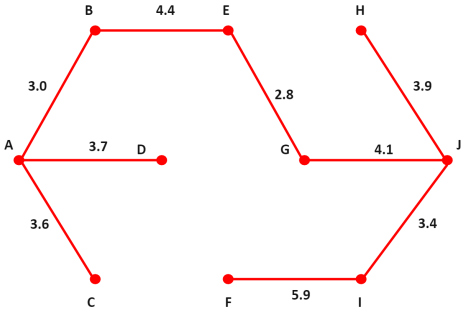
(a) (i) FI

(ii) DA

(b) (i) EG - AB - IJ - AC - AD - HJ - GJ - BE - FI

2.8 - 3 - 3.4 - 3.6 - 3.7 - 3.9 - 4.1 - 4.4 - 5.9

(ii) 34.8 m

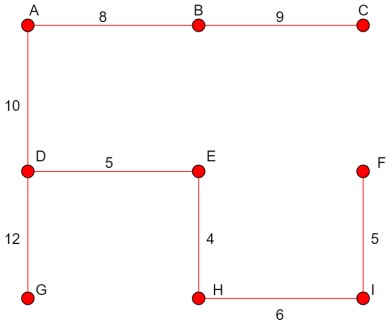


Question 4

(a) (i) (ii) A connected graph with no cycles.

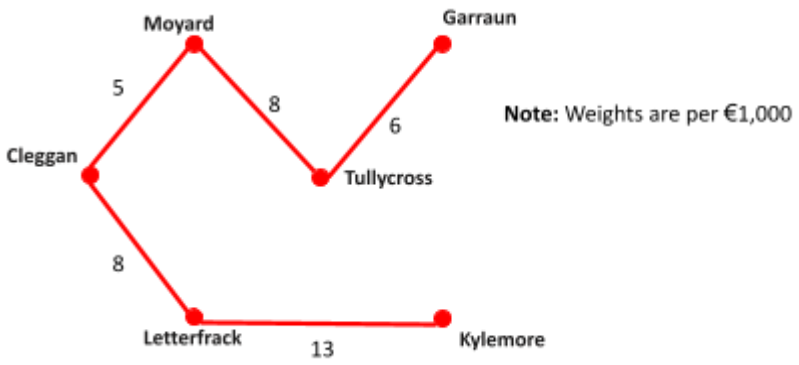
(ii) All nodes are connected and is also a tree.

(b) Weight = 59 *m*



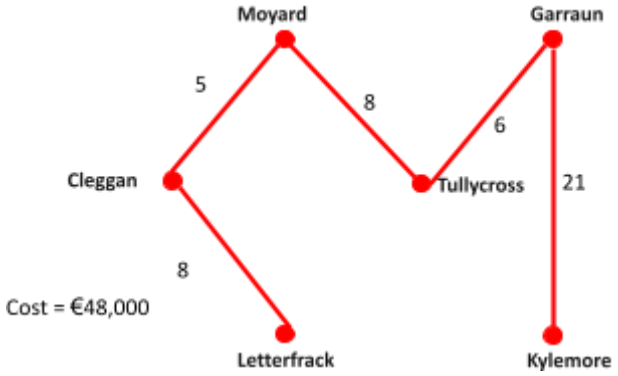
Question 5

(a) Minimum cost = €40,000



Note: Weights are per €1,000

(b) Start the tree with links between Tullycross and Garraun as well as Garraun and Kylemore and then apply Kruskal's algorithm.



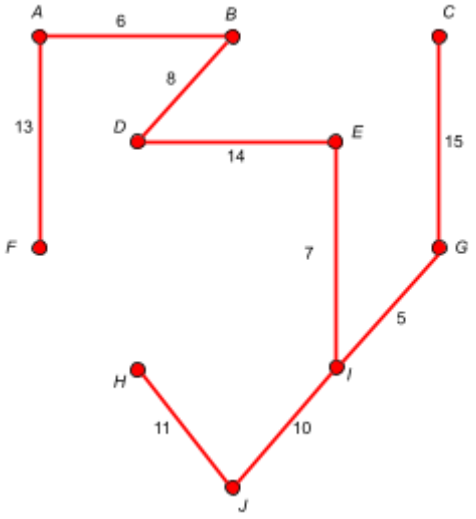
Question 6

(a) (i) 9

(ii) n - 1

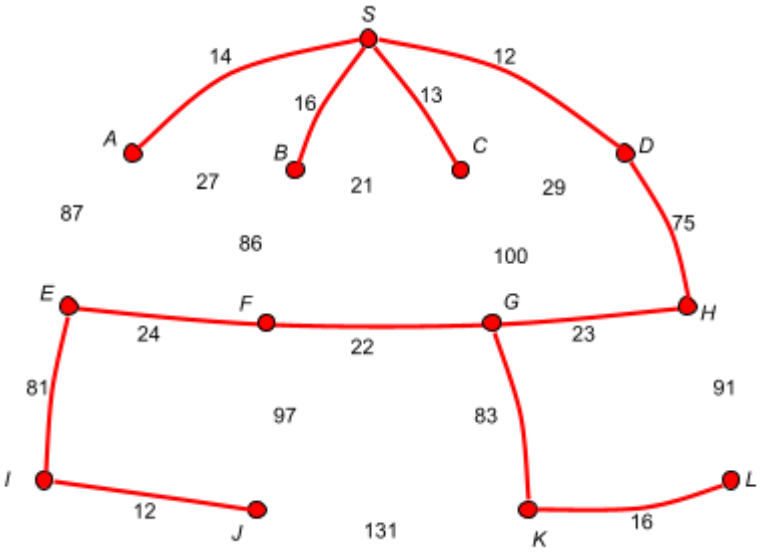
(b) (i)

(ii) 89 m



Question 7

(a) SD 12
SC 13
SA 14
SB 16
DH 75
HG 23
GF 22
FE 24
EI 81
IJ 12
GK 83
KL 16



(b) 391 *m*

(c)

(d) *GF* 7th (22)
HG 8th (23)