

Solutions: A collection of short problems on place value, integers, ordering and rounding.

Q1.

The totals of the top row and completed main diagonal are 30 and 39 respectively and therefore the 10 consecutive numbers in question must be 30 to 39 inclusive.

The number in the bottom right hand corner must be 1, 2, 8, 15 or 16. Taking 1 and 2 the completed diagonal would add up to less than 30 whilst 15 and 16 produces a number greater than 39. Hence 8 must go in the bottom right hand corner. It now follows that the last digit in the second row must be 15 since if 15 is placed in any one of the other three vacant squares, we get a total of 45 in the second column, 34 in the third column (same as the diagonal) and 47 in the third row (too big). We are now left with 1, 2 and 16. End third row must be 1 to make the column less than 39 (38) and the row total 33. This leaves 2 and 16 making the forth row total. 16 cannot go in column 2 as this makes a total greater than 39.

				39
4	5	7	14	30
6	13	3	15	37
11	12	9	1	33
10	2	16	8	36
31	32	35	38	34

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://rich.maths.org/2210/solution>

Q2.

The number may be divided up into 400 blocks of '12345'. The sum of the digits in each block is 15 and there are 400 blocks. Hence the sum of all 2000 digits is $400 \times 15 = 6000$.

Alternatively, the mean of each group of five digits is 3 and so the mean of the digits making up the number is 3. Therefore the sum is $2000 \times 3 = 6000$.

<http://rich.maths.org/2344/solution>

Q3.

Consider numbers whose first digit is 1. Looking at each possible value for the second digit we find 9 such numbers.

110, 121, 132, 143, 154, 165, 176, 187, 198.

Similarly there are 8 numbers starting with 2; 7 numbers starting with 3...

Lastly there is only one number starting with 9: 990.

Hence the total is $9+8+7+6+5+4+3+2+1 = 45$.

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/2347/solution>

Q4.

Both p and $-q$ are positive numbers. Hence $p + (-q)$ is the largest of the alternatives.

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/5762/solution>

Q5.

Firstly, there are 12 unit squares which contain an even number. Every 2×2 square in the diagram has entries which consist of two odd numbers and two even numbers and hence have an even total. There are 16 of these. Each 3×3 square in the diagram, however, has entries which consist of five odd numbers and four even numbers (giving an odd total), or four odd numbers and five even numbers (giving an even total). There are 4 of the latter: those with 8, 12, 14 or 18 in the centre. Every 4×4 square in the diagram has entries which consist of eight odd numbers and eight even numbers and hence have an even total. There are 4 of these. Finally, the full 5×5 square contains 13 odd numbers and 12 even numbers, giving an odd total. So the required number is $12+6+4+4$, that is 36.

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/5772/solution>

Q6.

13	6	11
8	10	12
9	14	7

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6215/solution>)

Q7.

Tailor.

Bilbo and Frodo are essentially counting 1 through 8 and then going round again (with Tinker = 1, Tailor = 2, etc.). So if Bilbo finished on Sailor = 4 and Frodo finished on Poor Man = 6, then together they would reach $4+6=10$. But 10 is 2 after 8, and 2 = Tailor.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6229/solution>)

Q8.

Tina gives Tony €0.70.

Let x be the amount Tony owes Tina (which will be negative if Tina owes Tony).

Tony owes Tina €0.40p: $x = 0.40$.

Tina borrows €0.50 from Tony: $x = -0.10$

Tony gives Tina €0.60: $x = -0.70$

So Tina owes Tony €0.70.

Q9.

10 ways

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/6238/solution>

1. $10 \times \text{€}0.01$
2. $5 \times \text{€}0.02$
3. $2 \times \text{€}0.05$
4. $1 \times \text{€}0.02 + 8 \times \text{€}0.01$
5. $2 \times \text{€}0.02 + 6 \times \text{€}0.01$
6. $3 \times \text{€}0.02 + 4 \times \text{€}0.01$
7. $4 \times \text{€}0.02 + 2 \times \text{€}0.01$
8. $1 \times \text{€}0.05 + 5 \times \text{€}0.01$
9. $1 \times \text{€}0.05 + 2 \times \text{€}0.02 + 1 \times \text{€}0.01$
10. $1 \times \text{€}0.05 + 1 \times \text{€}0.02 + 3 \times \text{€}0.01$

Q.10

Sophie Germain lived about $1831 - 1776 = 55$ years

Sonja Kowalevsky lived about $91 - 50 = 41$ years

Emmy Noether lived about $1935 - 1882 = 53$ years

So in order of lifetime, shortest-lived first:

Kowalevsky, Noether, Germain

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/6244/solution>

Q.11

2400m.

Between the 9 bus stops are 8 distances of 300m, so the total is $8 \times 300 = 2400$ m.

This problem is taken from the [UKMT Mathematical Challenges](#).

<http://nrich.maths.org/6246/solution>

Q.12

5.

The only way to get 87 is to get 13 questions correct and 2 wrong, leaving 5 questions unattempted.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6255/solution>)

Q.13

	Day Max	Night Min
Sunday	4°	-1°
Monday	5°	-5°
Tuesday	-1°	-3°
Wednesday	5°	

So on Wednesday evening the Maximum recorded temperature is 5° and the Minimum -5°

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6262/solution>)

Q.14

Kan must know for sure that Roo's cards are either both even or both odd. The only way he can be sure of this is if he's selected the three even cards leaving Roo only odd numbered cards. Therefore Kan has cards 2, 4 and 6 and the sum of his cards is 12.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6753/solution>)

Q.15

In the word DISCOVER the distinct pairs of letters DC, IE, SO, VR occur in reverse alphabetical order, so we must delete at least one of the letters in each pair, in other words remove at least four letters. In fact this is all we need to remove e.g. DISV is in alphabetical order.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6768/solution>)

Q.16

$\frac{4}{5} = \frac{12}{15}$ and $\frac{-2}{3} = \frac{-10}{15}$, so the number halfway between these is $\frac{1}{2} \left(\frac{-10}{15} + \frac{12}{15} \right)$ which is $\frac{1}{15}$.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/6787/solution>)

Q.17

Since $100=2 \times 2 \times 5 \times 5$, the only possible product of four *different* positive integers which equals 100 is $1 \times 2 \times 5 \times 10$. The sum of these integers is $1+2+5+10=18$.

(<http://nrich.maths.org/7132/solution>)

Q.18

Alberta is 52.

This problem is taken from the [UKMT Mathematical Challenges](#).

(<http://nrich.maths.org/7177/solution>)